What is...linear algebra?

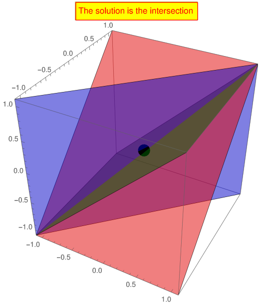
Or: Everything is linear.

The art of solving linear equations

$$\begin{cases} 0x + 1/2y - 1/2z = 0 & \text{(Red)} \\ -1/2x + 0y + 1/2z = 0 & \text{(Green)} \\ 1/2x - 1/2y + 0z = 0 & \text{(Blue)} \end{cases}$$

or equivalently

$$\left(\begin{array}{ccc|c} 0 & 1/2 & -1/2 & 0 \\ -1/2 & 0 & 1/2 & 0 \\ 1/2 & -1/2 & 0 & 0 \end{array}\right) \quad \leftrightsquigarrow$$



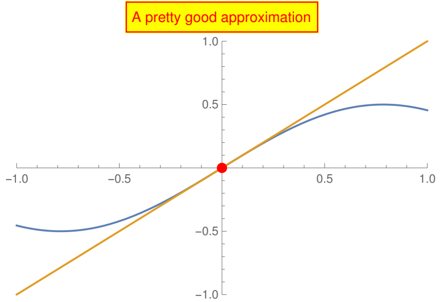
The need for a machine to solve linear equations grew out of...

- ...Cartesian geometry
- ...the geometry of lines, planes and hyperplanes
- ...that solving other types of equations (polynomial or even worse) is very hard

Linear algebra provides algorithms to solve linear problems

The art of first-order approximation

$$f(x) = \sin(x)\cos(x)$$
$$= \boxed{x} - \tfrac{2}{3}x^3 + \tfrac{2x}{15}x^5 + \ldots \quad \leftrightsquigarrow$$
$$= \boxed{\text{tangent}} + \text{REST}$$



The need for a machine to study linear maps grew out of...

- ...the observation that differentials are linear maps
- ...the need to have linear approximations of non-linear objects
- ...the need to have linear approximations of natural phenomena

Linear algebra provides tools to study properties of linear maps

The keywords – what linear algebra studies

- Vector spaces a.k.a. linear spaces
  - Coordinate spaces
  - Bases and dimensions
  - Angles and length of vectors
  - ...
- Linear maps / matrices
  - Eigenvectors and eigenvalues
  - Determinant and permanents
  - Jordan form
  - ...
- Linear geometry
  - Hyperplanes
  - Rotation, shearing, reflections
  - Systems of linear equations
  - ...

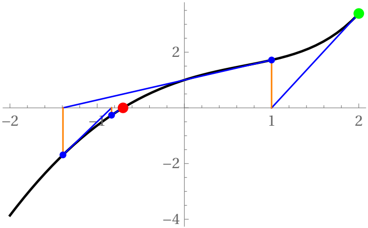
Application one – Newton approximation

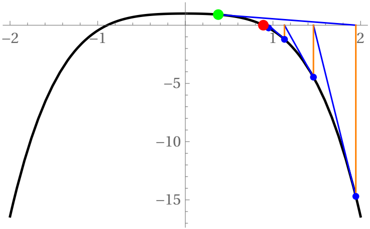
Question. Find x with $f(x) = 0$. Problem. For almost all f this is impossible.
Newton: Solve a linear problem instead.

(a) Given a point x_i and get tangent of f at x_i

(b) Find x_{i+1} with tangent$(x_{i+1}) = 0$ A linear equation!

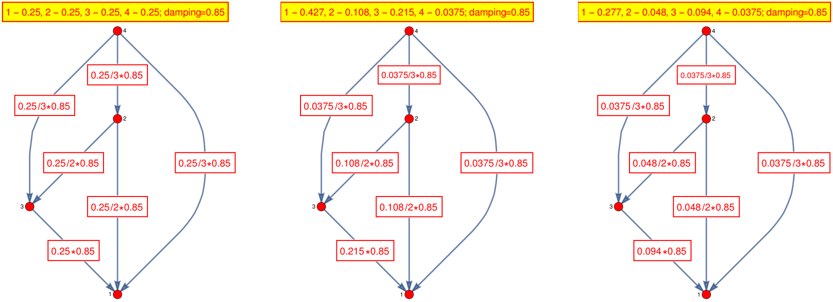
(c) Go back to (a) with x_{i+1}





Application two – PageRank by Brin–Page

The universe has four web pages with links given by:



The limit? An eigenvector problem!

$$\text{Solve: } 0.85 \cdot \begin{pmatrix} 0 & 1/2 & 1 & 1/3 \\ 0 & 0 & 0 & 1/3 \\ 0 & 1/2 & 0 & 1/3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} ?(1) \\ ?(2) \\ ?(3) \\ ?(4) \end{pmatrix} + \begin{pmatrix} (1-0.85)/4 \\ (1-0.85)/4 \\ (1-0.85)/4 \\ (1-0.85)/4 \end{pmatrix} = \begin{pmatrix} ?(1) \\ ?(2) \\ ?(3) \\ ?(4) \end{pmatrix}$$

Thank you for your attention!

I hope that was of some help.