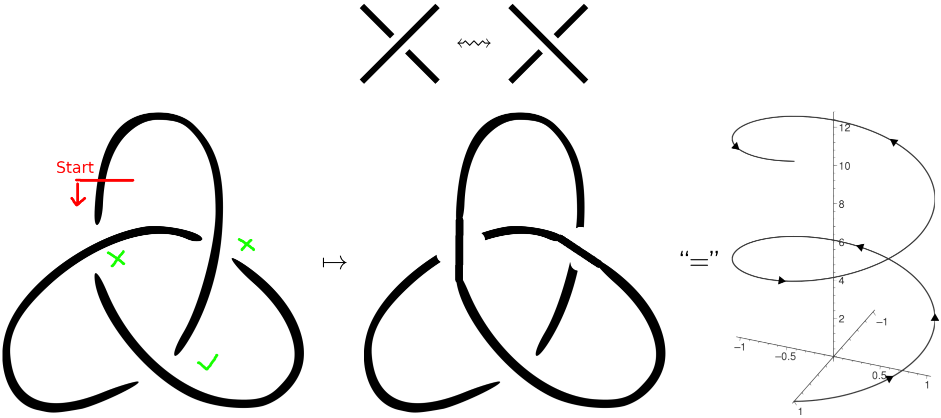
What is...the Alexander polynomial?

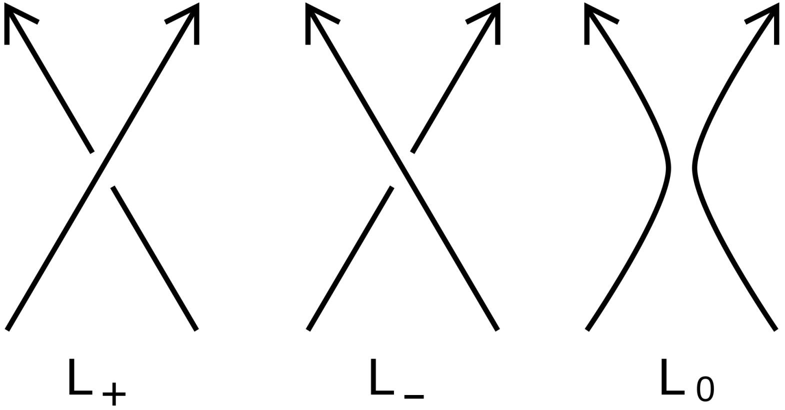
Or: Alexander–Conway skein calculus

Flipping crossings



- Theorem The flipping crossing operation can trivialize every knot
- Proof Produce a helix
- Idea Use this somehow

Skein theory



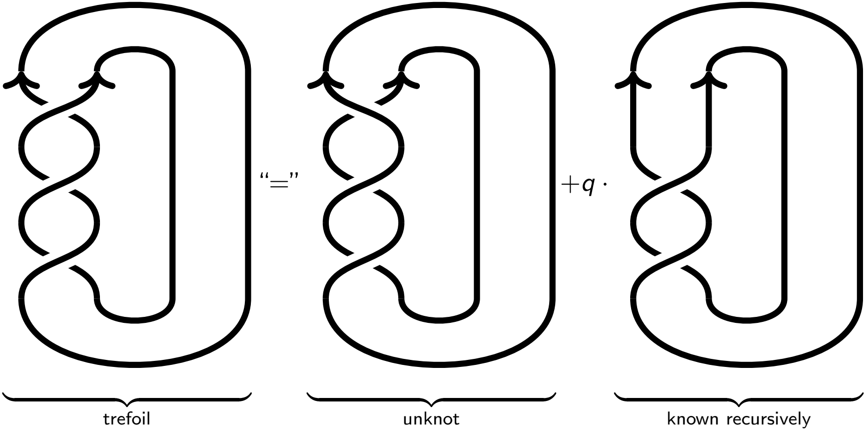
- Skein theory Force a local relation onto crossings, *e.g.*

$$\nabla(L_+) - \nabla(L_-) = q\nabla(L_0)$$

- Either L_+ or L_- are easier, and L_0 clearly is easier $\Rightarrow$ should give a recursion

A recursively defined polynomial

Computing ∇ of the trefoil



- Say $\nabla(\text{unknot}) = 1$ and $\nabla(\text{easier}) = q$
- Then $\nabla(\text{trefoil}) = \nabla(\text{unknot}) + q\nabla(\text{easier}) = 1 + q^2$

For completeness: A formal statement

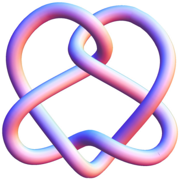
The polynomial $\nabla(\_) \in \mathbb{Z}[q]$ defined by

(i) $\nabla(\text{unknot}) = 1$ Normalization

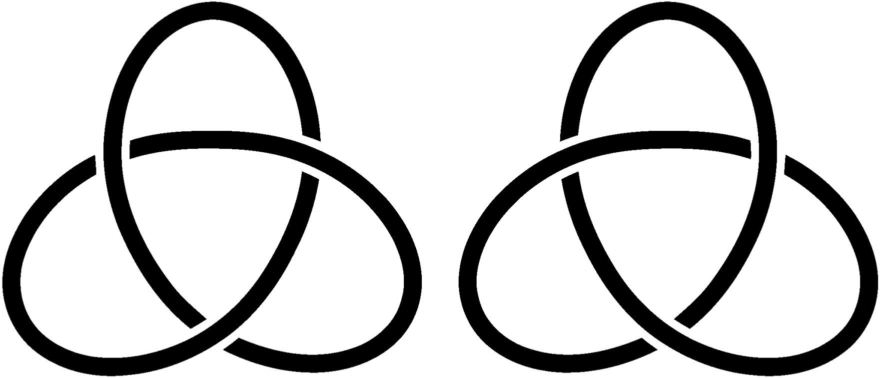
(ii) $\nabla(L_+) - \nabla(L_-) = q\nabla(L_0)$ recursion

is a well-defined knot invariant

- $\nabla(\_)$ is called the Conway polynomial
- Alexander original polynomial $\Delta(\_) \in \mathbb{Z}[q, q^{-1}]$ is a rescaling of $\nabla(\_)$
- The Alexander polynomial is relatively strong, *e.g.*

$\nabla\Bigg($  $\Bigg) 1 - q^2 \neq 1 - 2q^2 = \nabla\Bigg($  $\Bigg)$

Left = right-handed trefoil? No idea...



- The left-handed trefoil L has $\nabla(L) = 1 + q^2$
- The right-handed trefoil R has $\nabla(R) = 1 + q^2$
- Thus, we still can't tell them apart

Thank you for your attention!

I hope that was of some help.