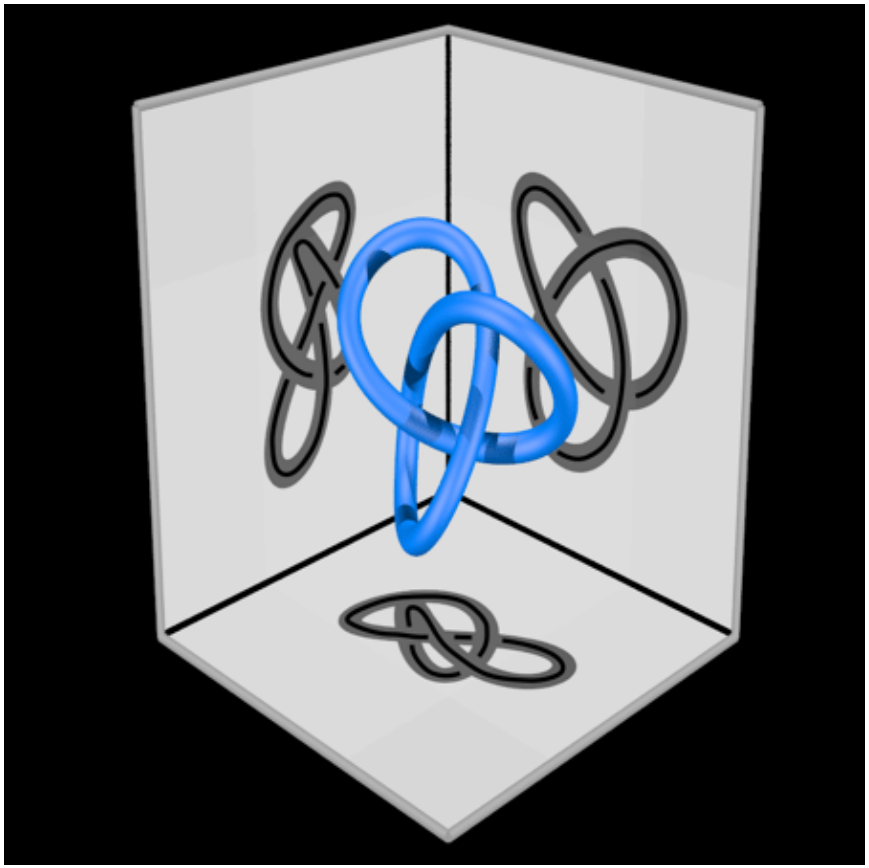
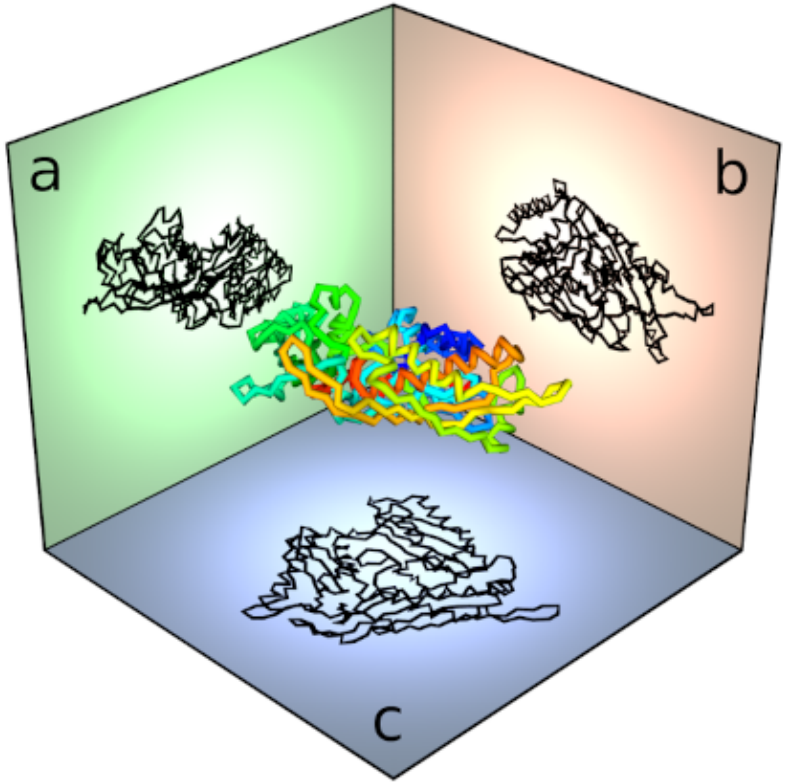
What are...knots and DNA?

Or: Applications 1 (topology in biochemistry)

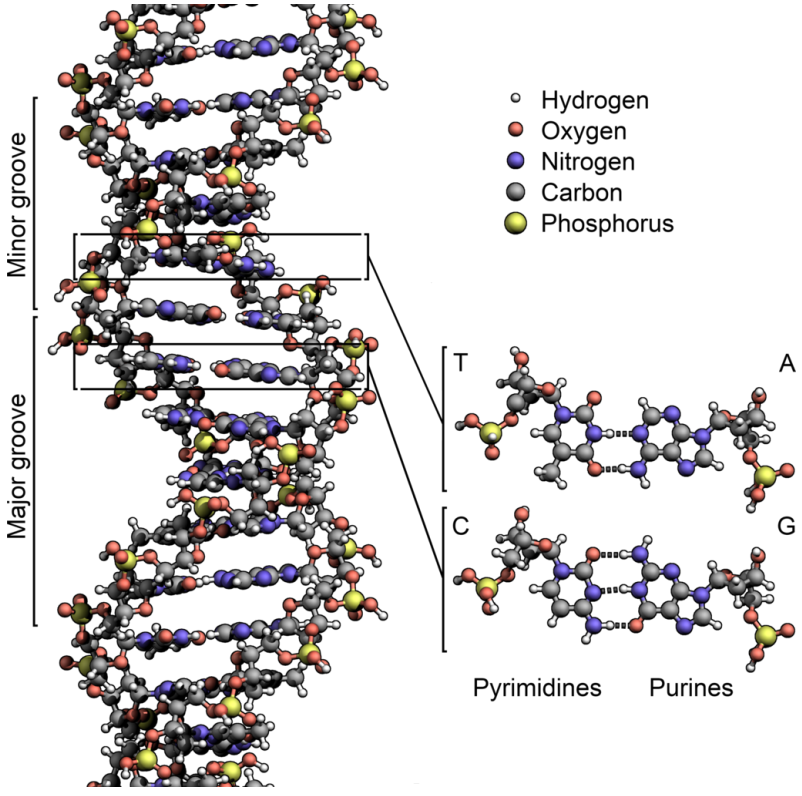
Reminder: Knots

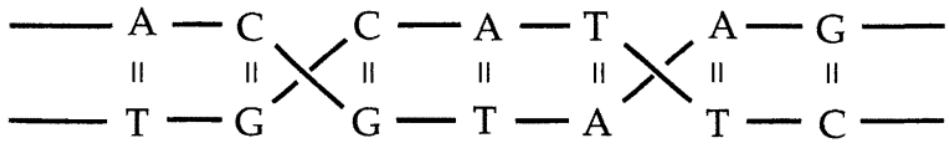




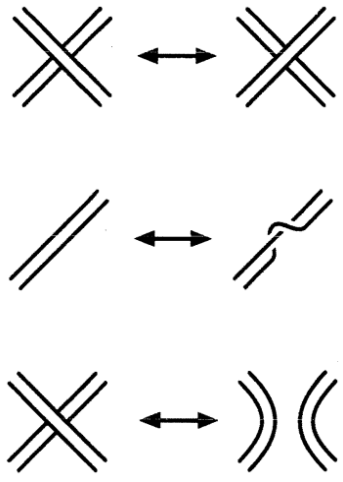
- Knots are studied by projections to the plane Shadows
- The main point are knot invariants (like polynomials *etc.*)
- Knots play the key role in everything low-dimensional

Enter, DNA (deoxyribonucleic acid)



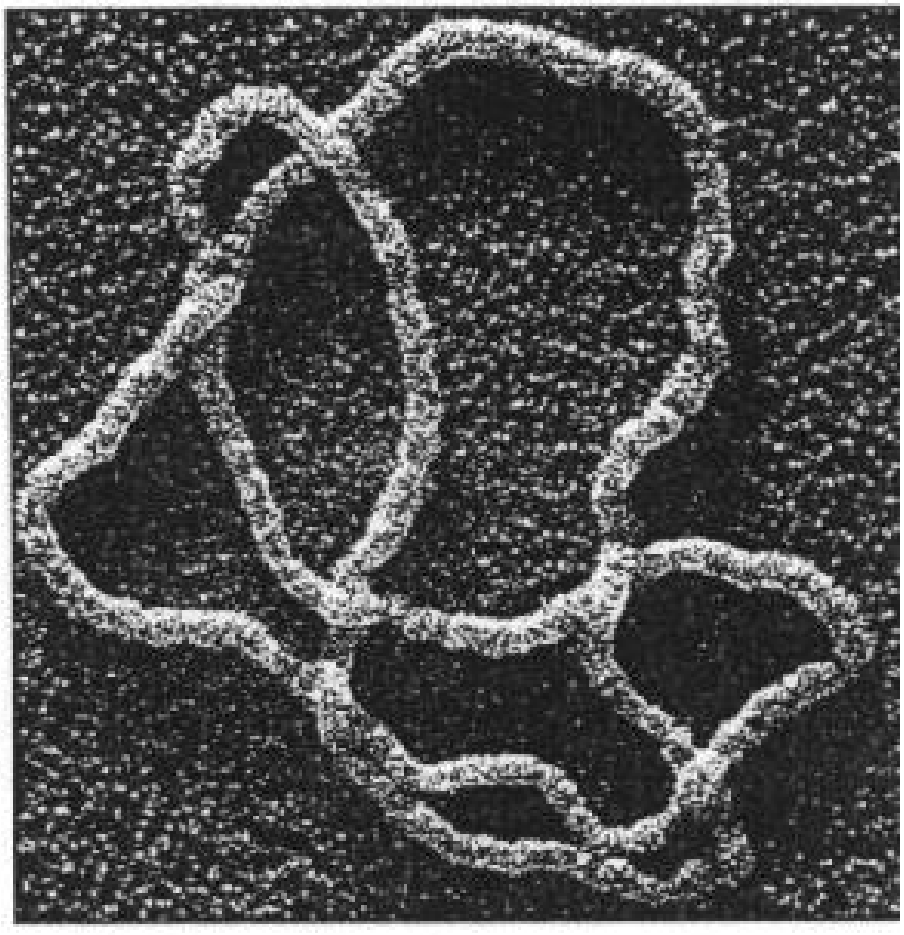


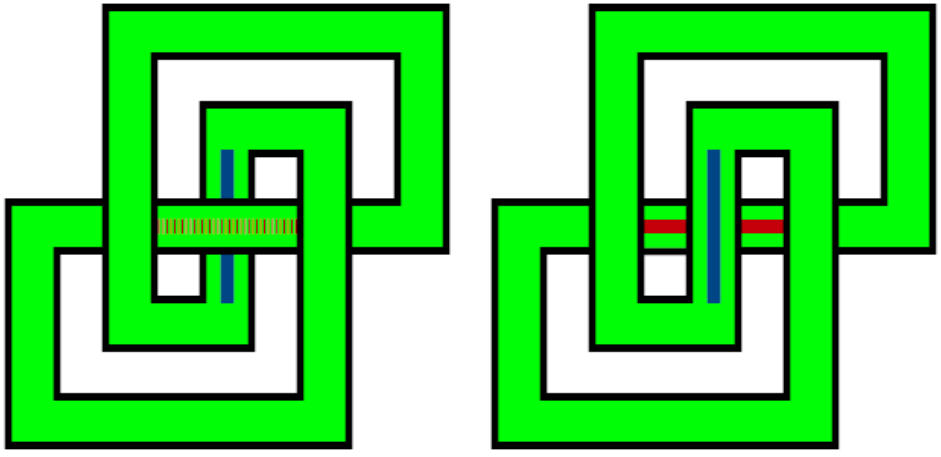
The DNA double helix.



- Deoxyribonucleic acid (DNA) is a molecule that is formed by pairs of long molecular strands that are bonded together by ladder rungs and that spiral around each other Double helix – already some braid
- Stuffing that into the tiny nucleus of a cell gives a tangled mess
- To untangle, enzymes manipulate the DNA topologically

Topology meets biochemistry





- DNA is knotted and enzymes ‘unknot’ DNA before *e.g.* replication
- Question What knots appear?
- Question What is the unknotting number of DNA?

For completeness: A comparison

Biochemists use invariants (Jones polynomial, Khovanov homology *etc.*) to detect knottedness and knot tables to determine properties

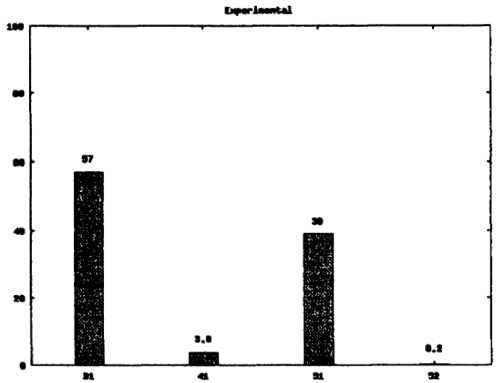


Fig. 3 The results of the Distribution of knots 3_1, 4_1, 5_1 and 5_2 by biological experiments [2]

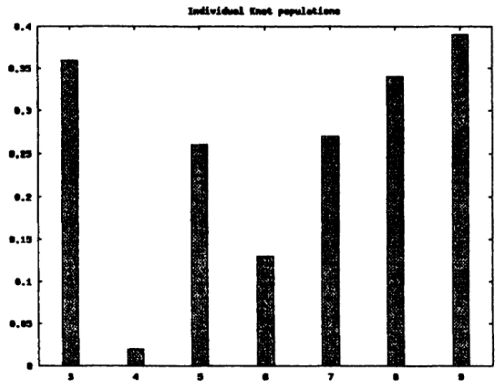
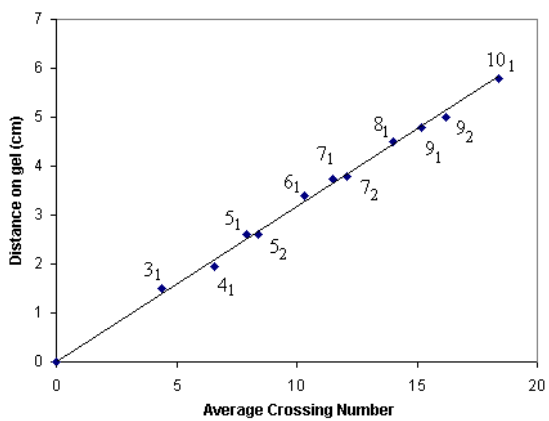


Fig. 4 Quantification of the individual knot populations of three to nine crossings

Mathematicians should use knowledge from biochemistry to study knots:

crossing number of DNA in gel :



How random is DNA?

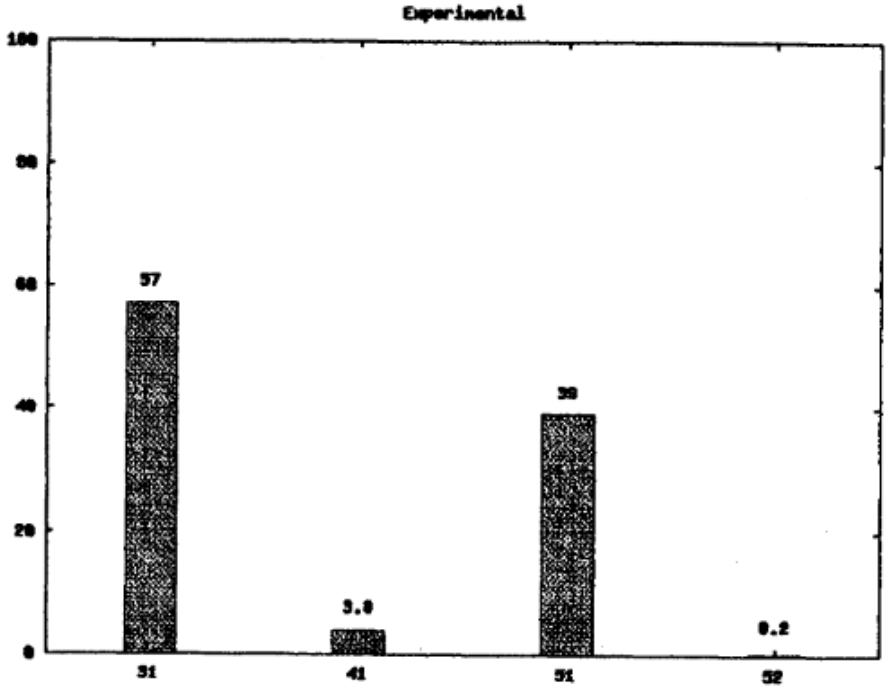


Fig. 13 The results of biological experiments by J. Arsuaga et. al. [2]

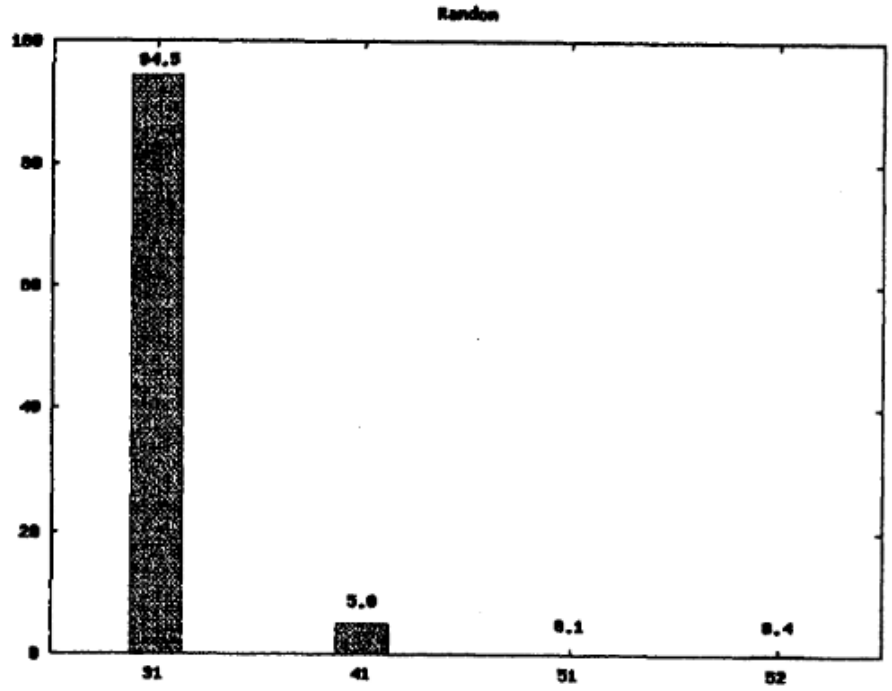


Fig. 14 Distribution of the knots 3_1, 4_1, 5_1 and 5_2 generated at random

- A randomly created nontrivial knot is essentially always a trefoil
- DNA knots are quite often other knots, *e.g.* 5_1
- DNA knots are not random $\Rightarrow$ there is some mechanism important for life that causes biases

Thank you for your attention!

I hope that was of some help.