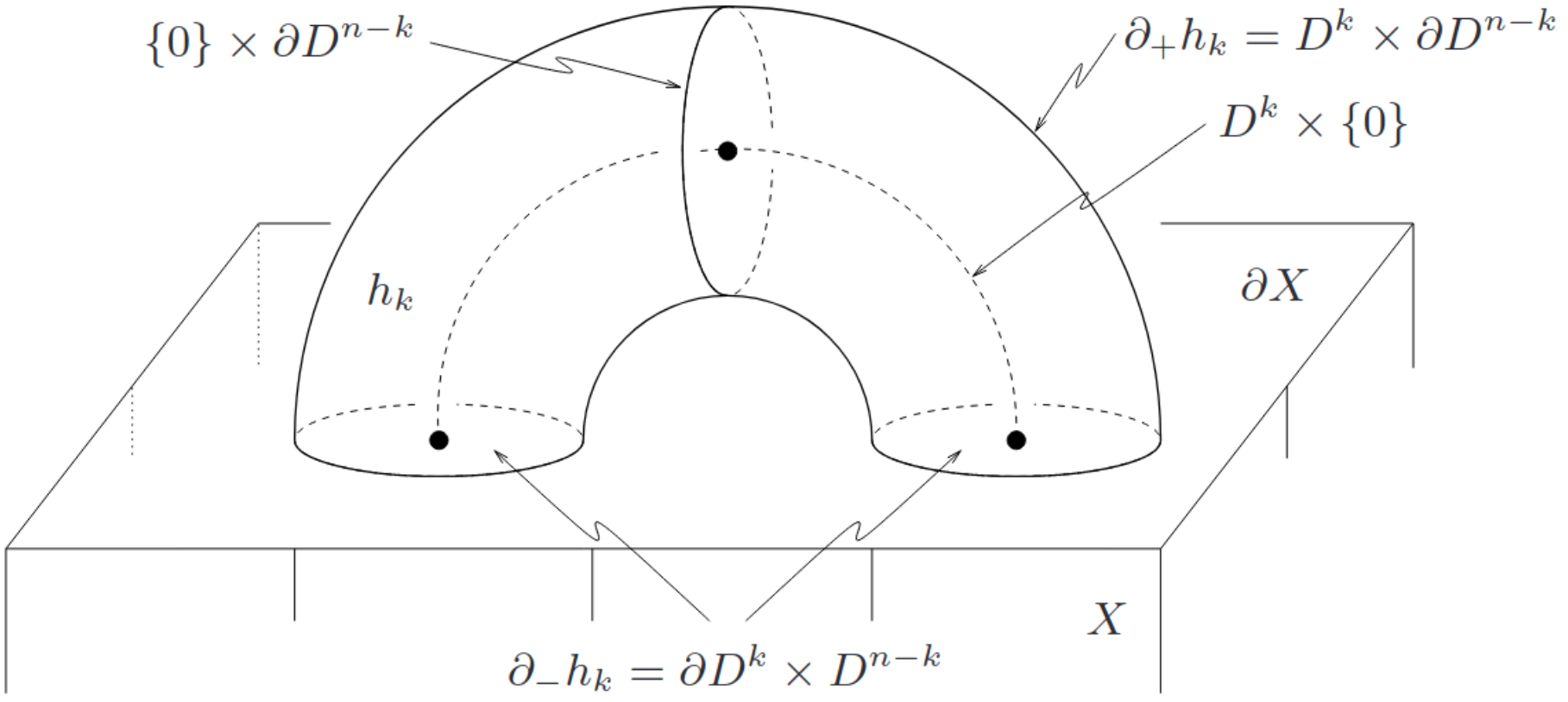
What is...4d Kirby calculus?

Or: Knots and four manifolds, part 1

Attaching handles

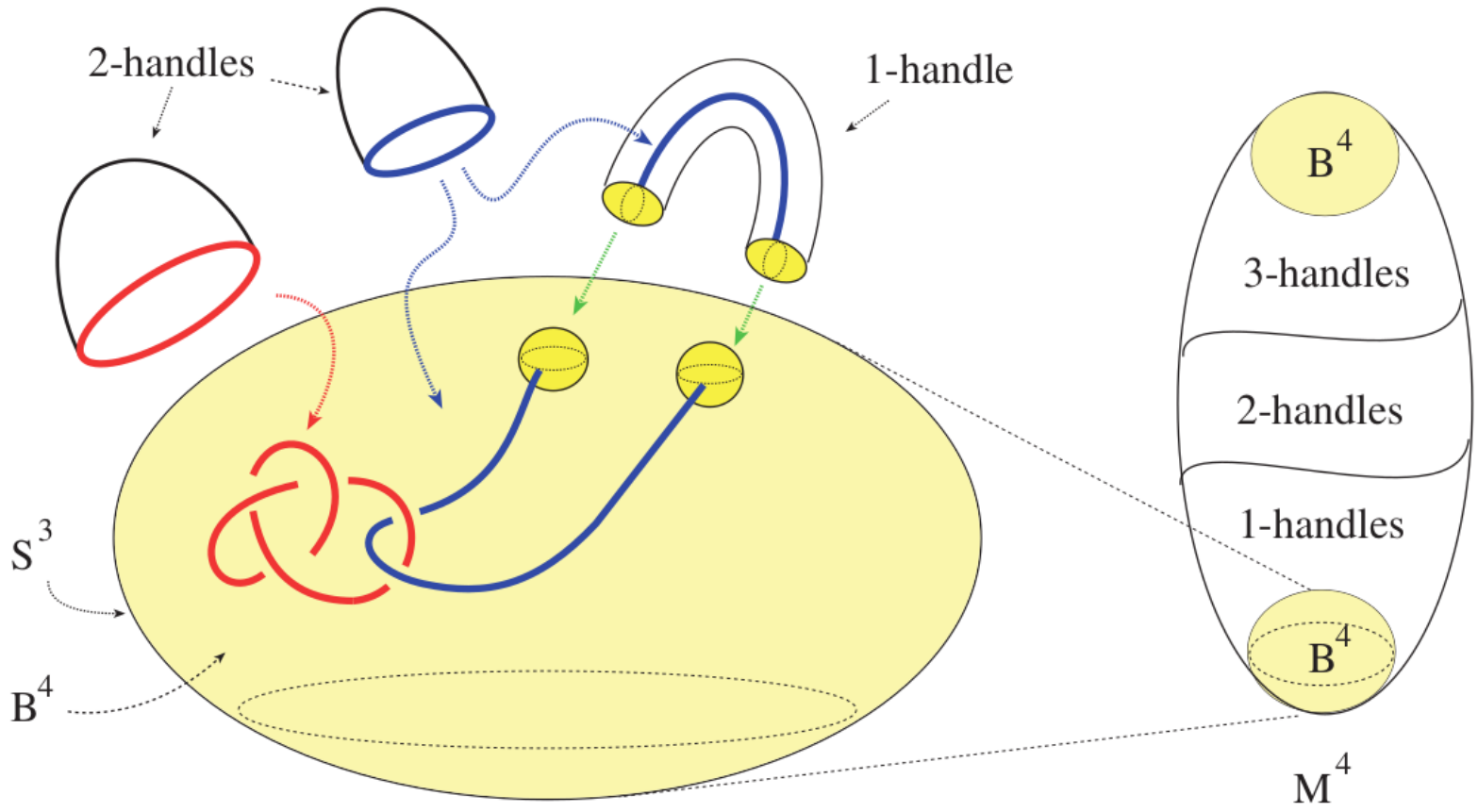


- Handle decomposition $\leftrightsquigarrow$ attaching k handles to an n mfd X
- k handle $= h^k = D^k \times D^{n-k}$
- Attaching depends on k, dimension is always n

1+2=3

- To construct 4mfds we a priori need 0,1,2,3,4 handles
- 0 and 4 handles are just "opening-closing" so we can ignore them
- Theorem 3 handles are determined by 1 and 2 handles
- Hence, we effectively need only 1 and 2 handles

Knots again!

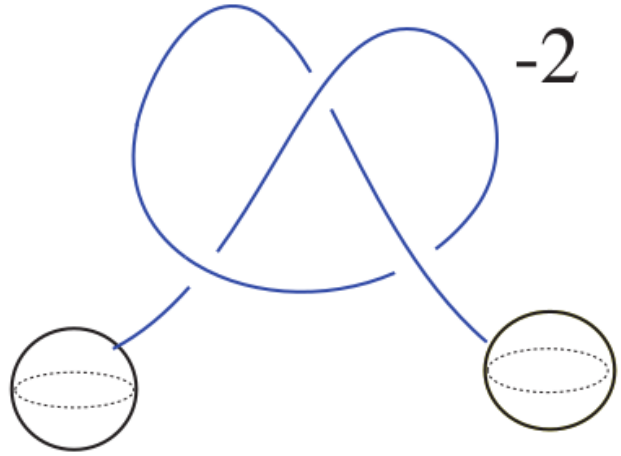


- The attaching sphere for a 1 handle is δD^1 Two points/two balls
- The attaching sphere for a 2 handle is δD^2 A (framed) knot
- The above picture is a Kirby presentation of a 4mfd

For completeness: A formal statement

Any closed orientable smooth/piecewise-linear 4-mfd admits a Kirby presentation

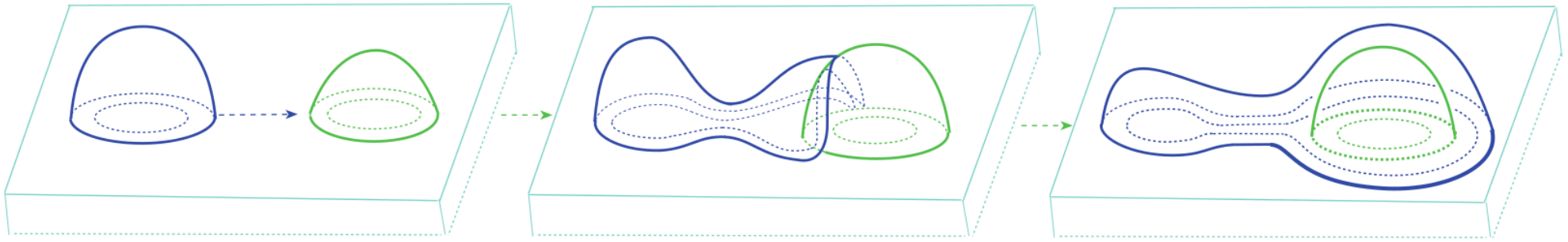
- Note that one can stack 2 handles on 1 handles
- The diagram you see in a Kirby presentation is a 4d Kirby diagram

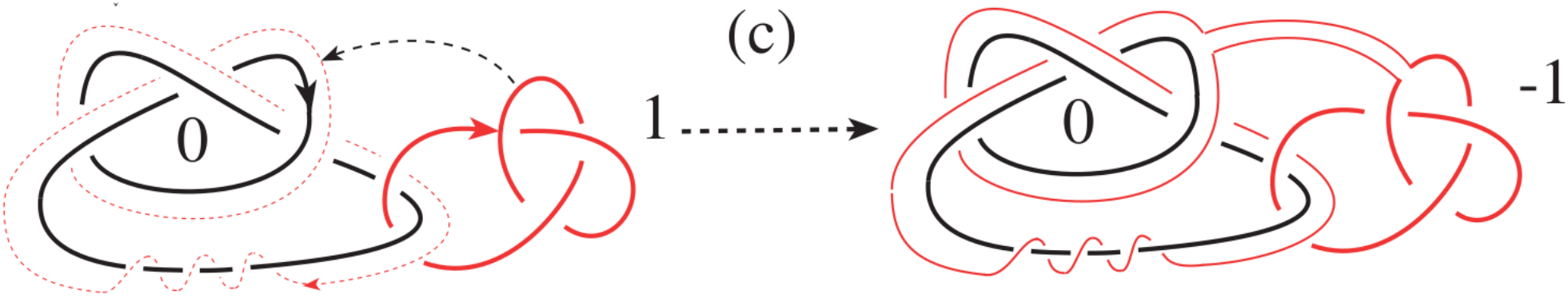


4d Kirby diagrams consists of pairs of balls for 1 handles and of framed=numbered knots for 2 handles

- There are some relations among 4d Kirby diagrams, e.g. handle slides

The handle slides again





- One can slide a k handle over another r handle when $r \leq k$
- Top Sliding a 2 handle over a 2 handle
- Top Sliding a 2 handle over a 2 handle in 4d Kirby diagrams

Thank you for your attention!

I hope that was of some help.