What is...a Heegaard diagram?

Or: Handle diagrams

Heegaard splitting

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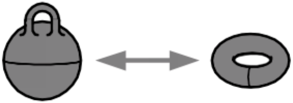
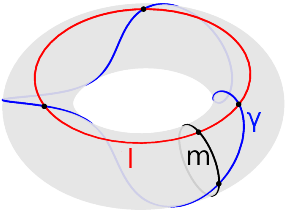


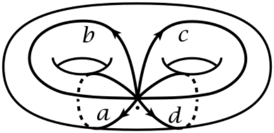
- Heegaard splitting $M = H \cup H'$ for handlebodies H, H' with $H \cap H' = \delta H = \delta H'$
- Exists for all closed orientable 3mfds
- Goal Describe it combinatorially

Gluing along the boundary

$g = 1$, meridian m:

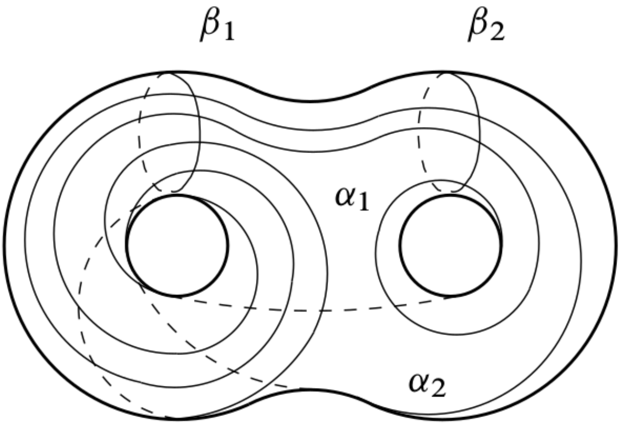


$g = 2$, meridians a,d:



- The boundaries δH and $\delta H'$ are surfaces S of genus g
- Similarly as for Dehn surgery the gluing along these boundaries is determined by where the "meridians" go
- It thus suffices to specify curves $\beta_1, \ldots, \beta_g$ for $\delta H'$, or split into two and we specify $\alpha_1, \ldots, \alpha_g$ for δH as well

Heegaard diagrams



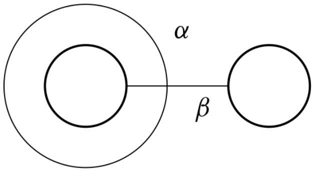
- A Heegaard diagram consists of two g-tuples of curves α_i, β_i in S
- Each g-tuple must consist of disjoint simple closed curves whose homology classes are linearly independent
- The α_i, β_i are the attaching curves for δH and $\delta H'$

For completeness: A formal statement

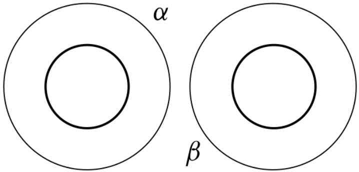
Heegaard diagrams describe closed orientable 3mfds

For $g = 1$ we can also draw diagrams in a plane minus two discs; to get back S compactify by one point at infinity and glue the two boundary circles together

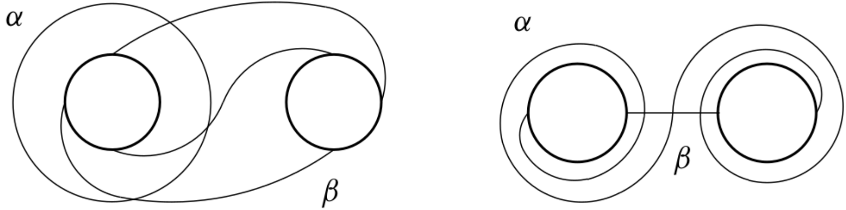
- Here is a Heegaard diagram of S^3:

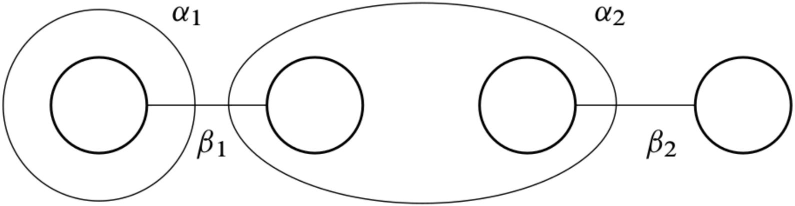


- Here is a Heegaard diagram of $S^1 \times S^2$:



More Heegaard diagrams





- **Above** Two Heegaard diagrams of $L(3,1)$
- **Below** A Heegaard diagram of genus $g = 2$

Thank you for your attention!

I hope that was of some help.