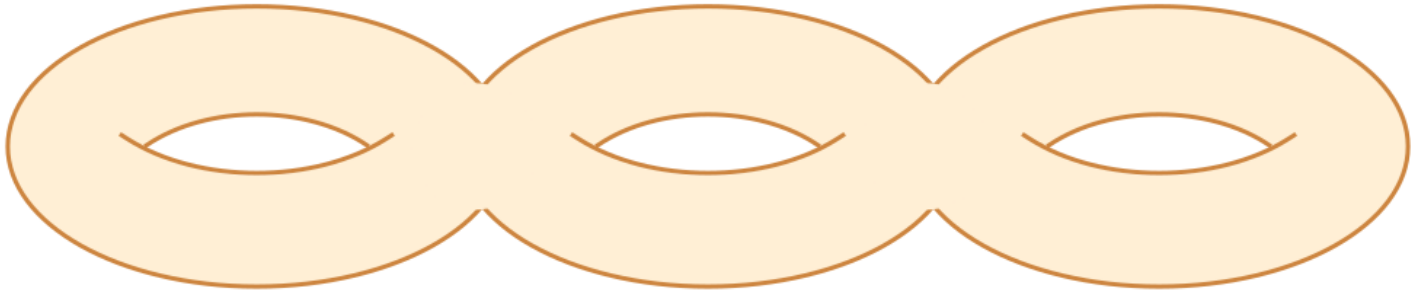
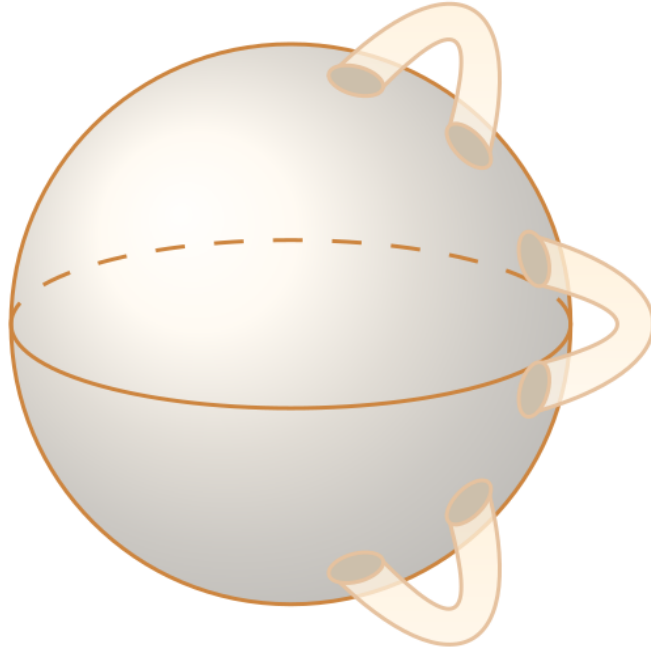
What is...a Heegaard splitting?

Or: Attaching handles

The 2d case



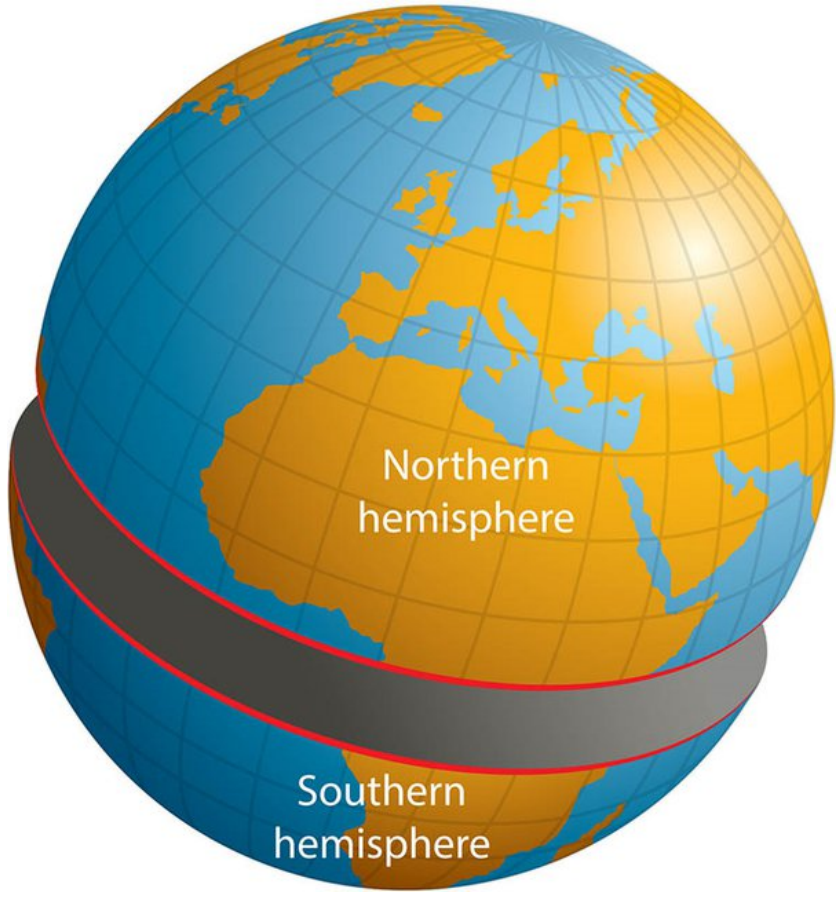


- Recall that every closed orientable 2mfd is obtained by gluing handles to S^2
- Goal Find a higher dimensional analog

Handlebodies

- Attaching handles to D^3 makes sense as well and we get 3d handlebodies
- These are the same beast as before but filled and not hollow

Heegaard splitting

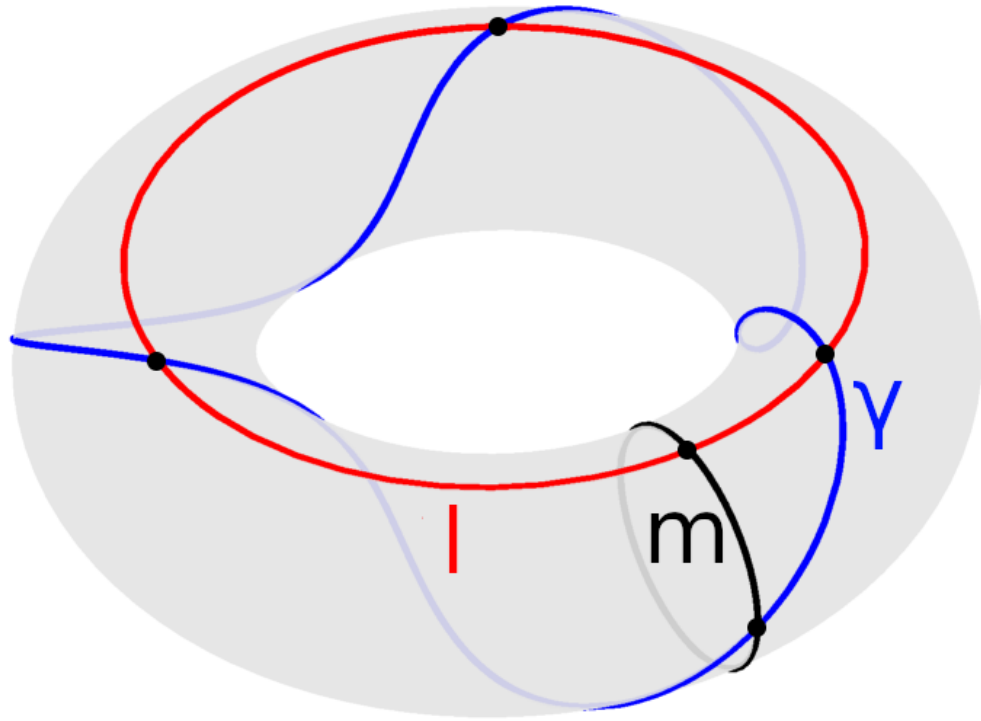


- Write a 3mdf $M = H \cup H'$ for handlebodies H, H' with $H \cap H' = \delta H = \delta H'$; this means M is glued together along H, H'
- This is called a Heegaard splitting
- Example S^3 is D^3 and D^3 glued together along S^2

For completeness: A formal statement

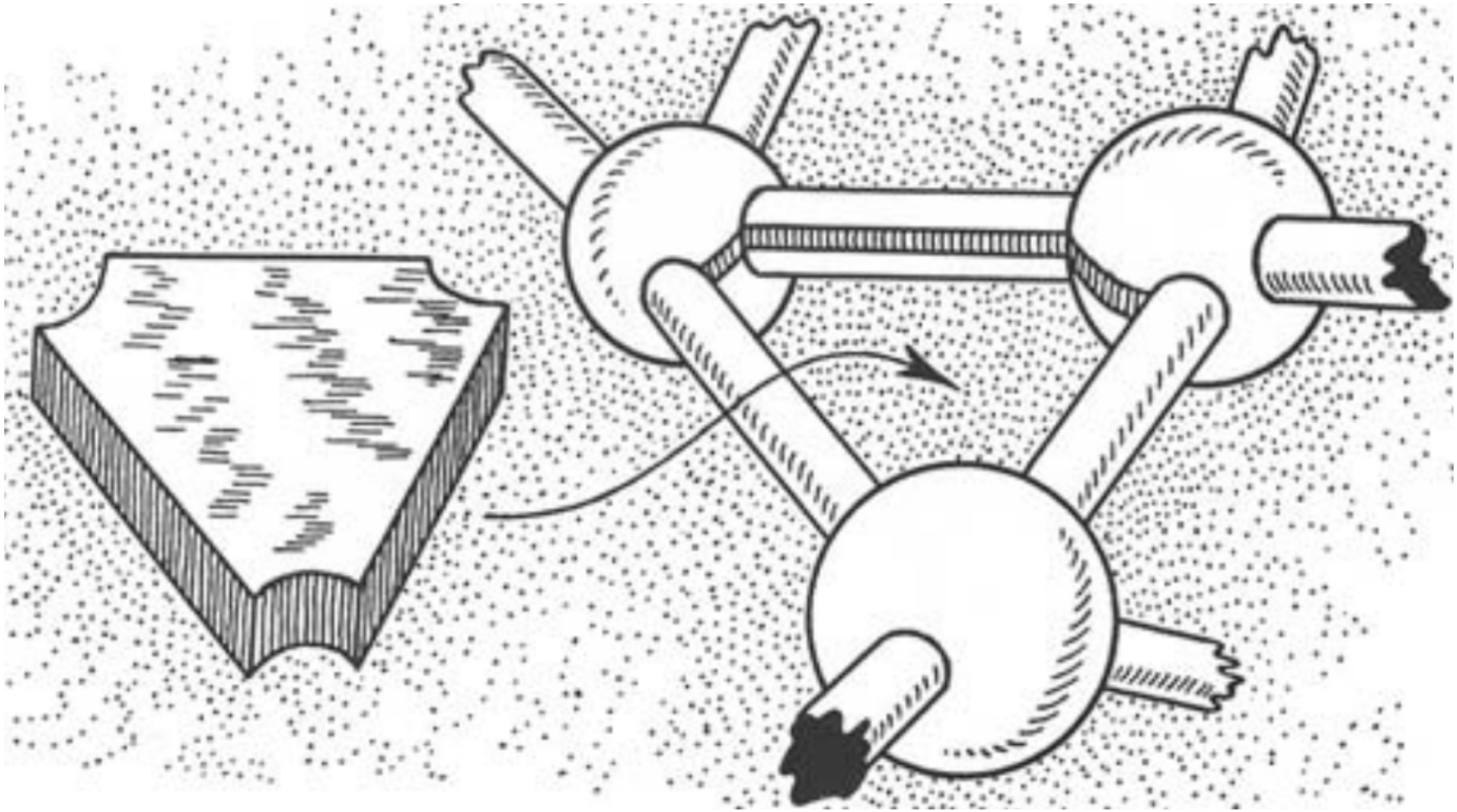
Any closed orientable 3-mfd admits a Heegaard splitting

- Heegaard splittings are similar to Dehn surgery as we will see



- In the next video we will see more technology to study Heegaard splittings

Proof



- Take a triangulation of M and replace vertices by balls, edges by cylinders, each side of a tetrahedron by a "plate", and each tetrahedron by a ball
- The union of the vertex balls and the cylinders is a handlebody, and so is the union of the tetrahedra balls and plates

Thank you for your attention!

I hope that was of some help.