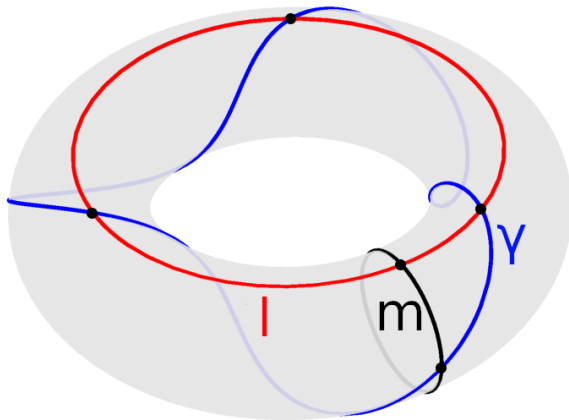


What is...Kirby calculus?

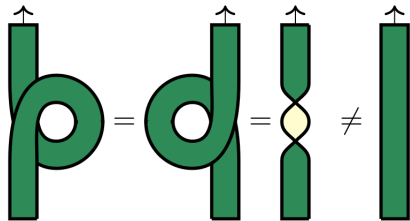
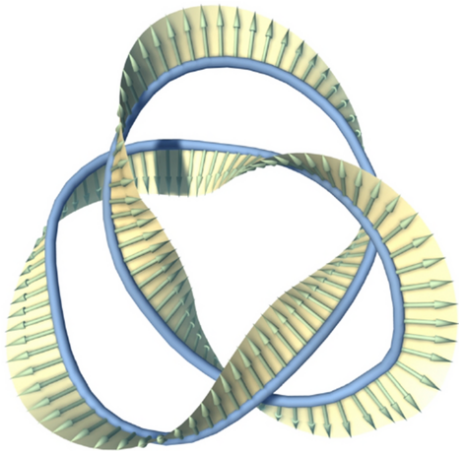
Or: Knots and three manifolds, part 2

Like the Reidemeister theorem



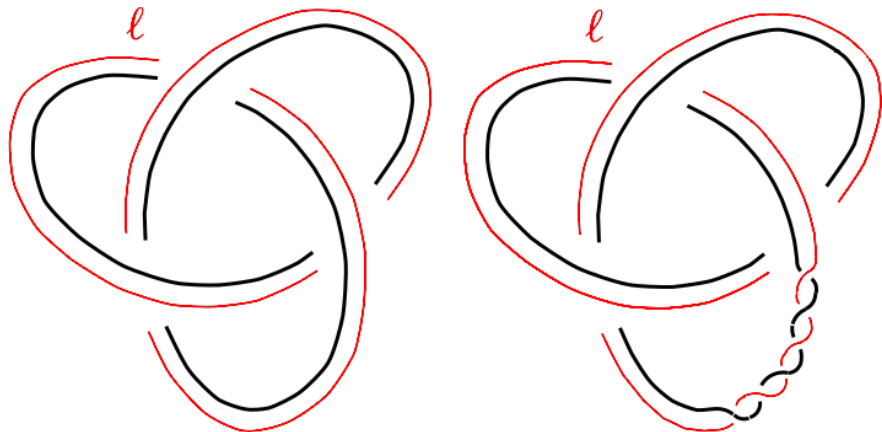
-
- ▶ Every closed, orientable, connected 3mfd can be obtained by **gluing meridians**
 - ▶ The process is **determined** by p/q -labeled knots with $q = \pm 1$
 - ▶ **Kirby calculus** identifies the relations among these

Framed knots



-
- ▶ What one needs to consider are framed knots
 - ▶ Framed knot \iff embedded ribbon; framing = the (signed) number of twists
 - ▶ We can think of these as knot diagrams with modified Reidemeister 1 move

Why framing?

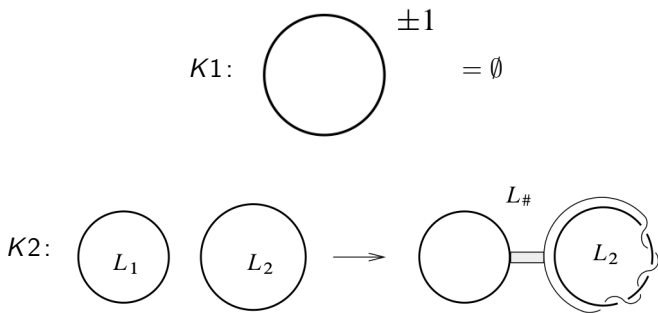


-
- ▶ A knot diagram picture is a bit misleading as we are really looking at **tori**
 - ▶ The choice of longitude might **change** the surgery
 - ▶ To encode that we think of the knots as ribbons

For completeness: A formal statement

A 3mfd obtained via surgery from S^3 are the same if and only if :

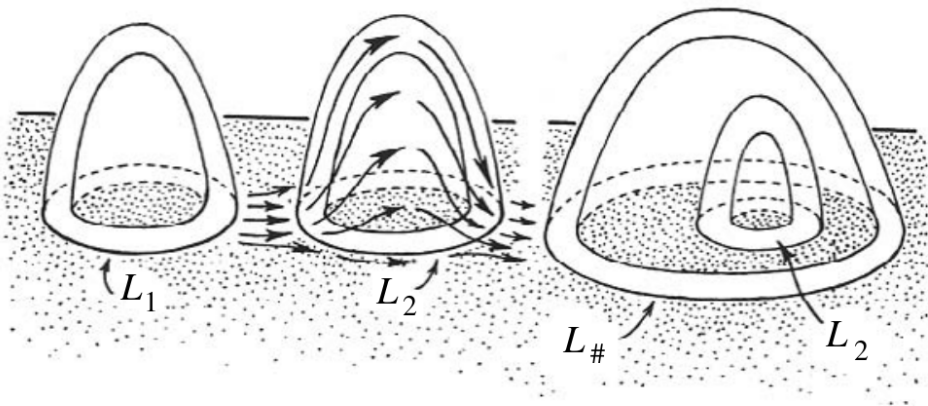
- (i) The surgery knots are the same as framed knots
- (ii) The surgery knots are related by the two Kirby moves :



where the new component $L_{\#}$ has framing $fr(L_1) + fr(L_2) + 2lk(L_1, L_2)$

“the same” = homeomorphic by an orientation preserving homeomorphism

The Kirby moves hold



- ▶ All ± 1 surgeries along unknots give back S^3 so the first Kirby move holds
- ▶ The second Kirby move is a handle slide
- ▶ The main point is that these moves are sufficient

Thank you for your attention!

I hope that was of some help.