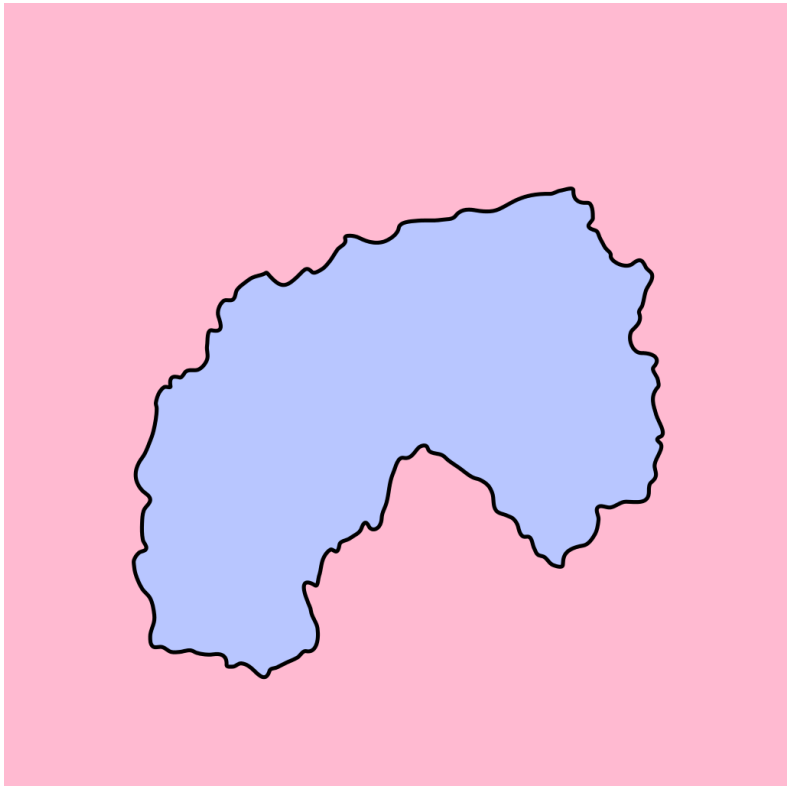
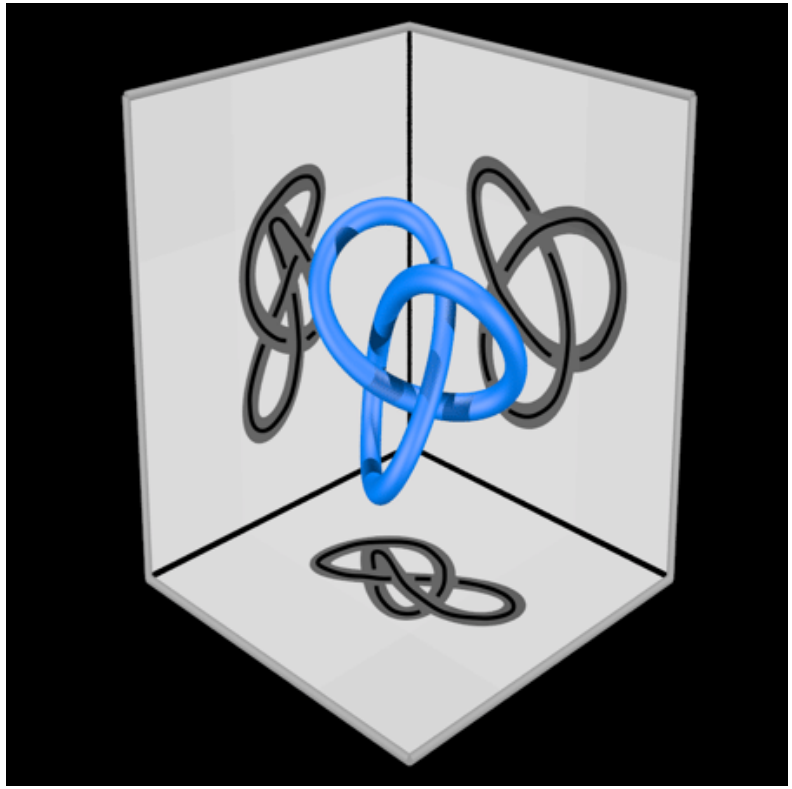
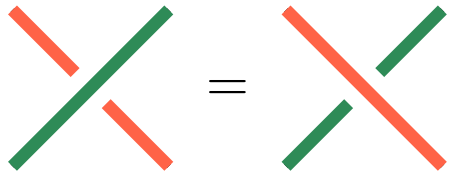
What are...knotted surfaces?

Or: Codimension 2

Knots in 2d, 3d and 4d

$2d$: 

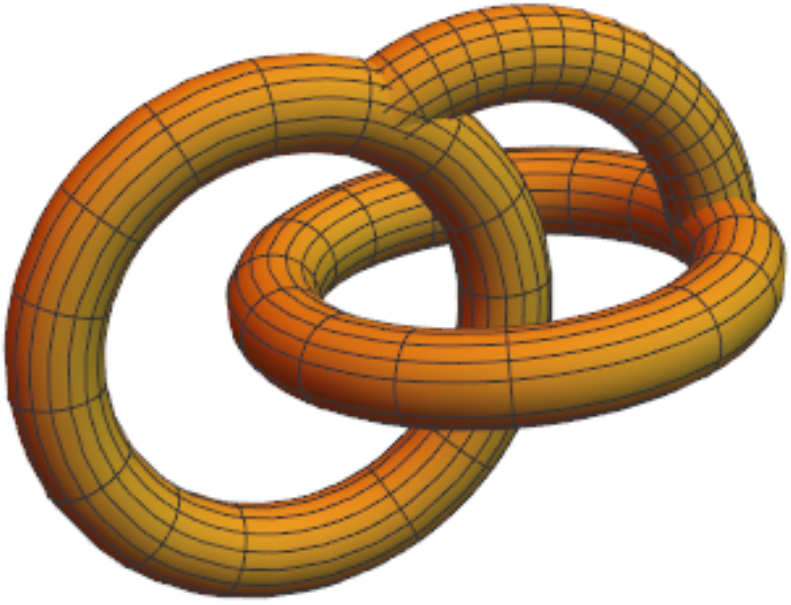
, $3d$: 

$4d$: 

- Knots in 2d are boring, knots in 4d can be undone, knots in 3d are great
- Note that "3d (ambient space) - 1d (knot) = 2d" Codimension 2

Knotting surfaces in 3d

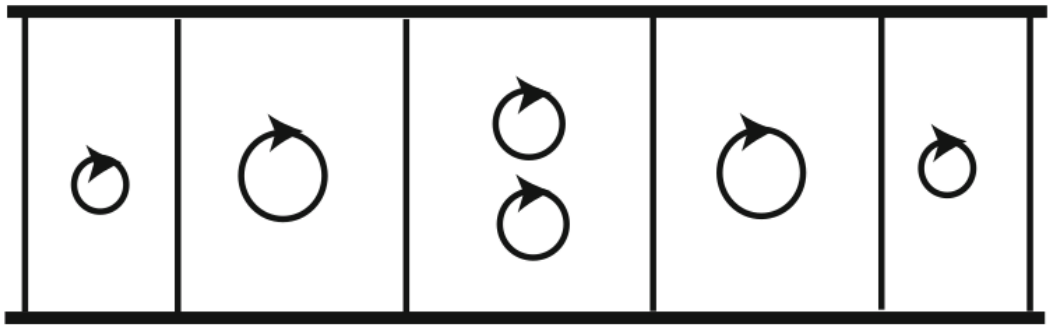
Knotted!



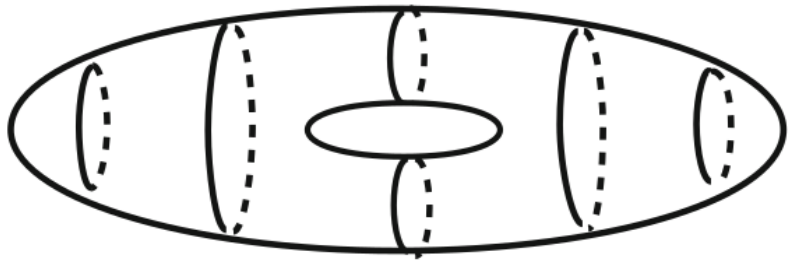
No idea how to knot this...

- One can knot some surfaces in 3d
- But that doesn't work for all surfaces
- This is not what I would like to discuss Codimension 2 is more fun

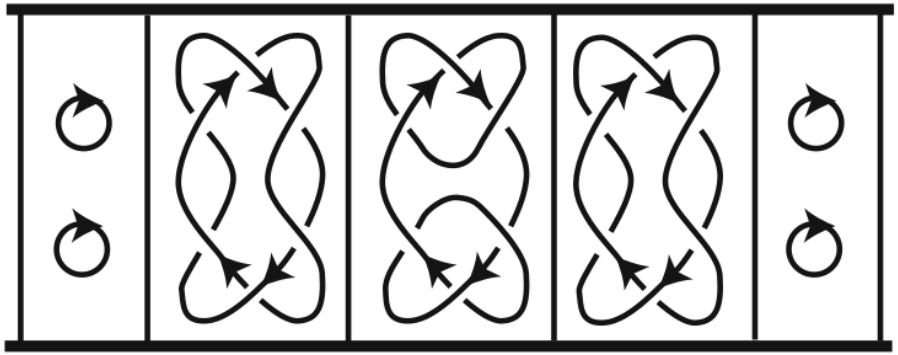
Knotting surfaces in 4d



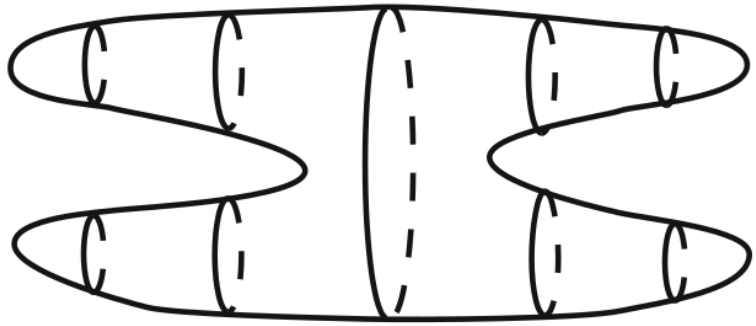
t=-2 t=-1 t=0 t=1 t=2



t=-2 t=-1 t=0 t=1 t=2



t=-2 t=-1 t=0 t=1 t=2



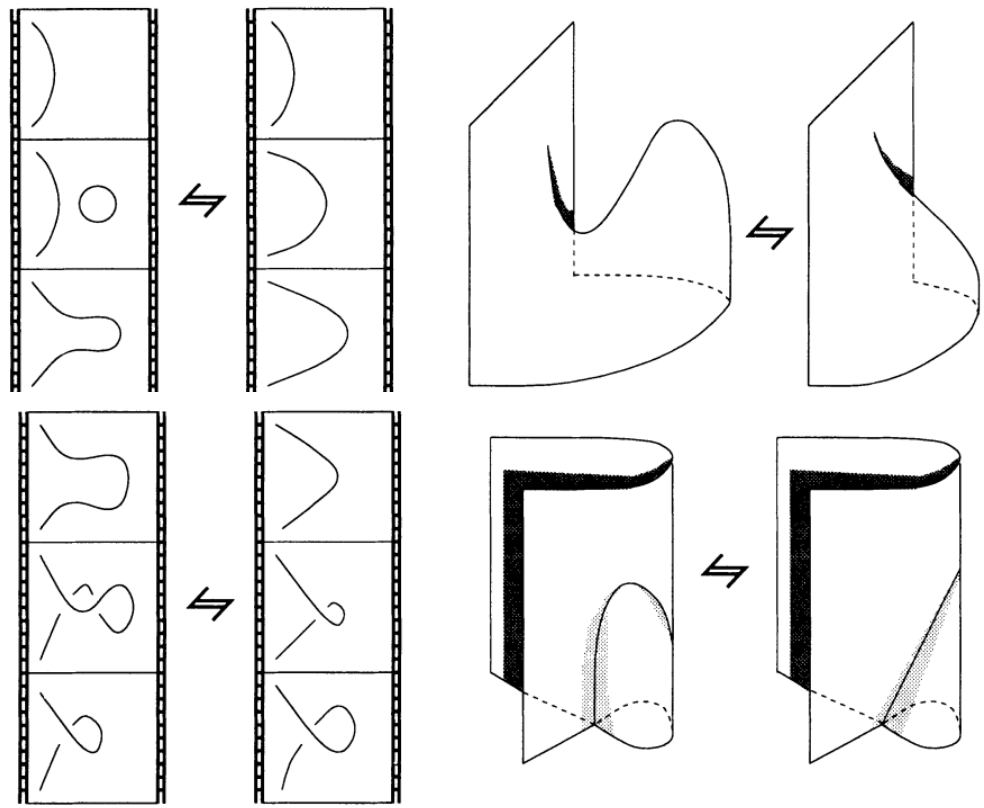
t=-2 t=-1 t=0 t=1 t=2

- A knotted surface in 4d can imagined as a movie
- Every frame is a knot in 3d
- Playing the movie (= time) makes everything 4d

For completeness: A formal definition

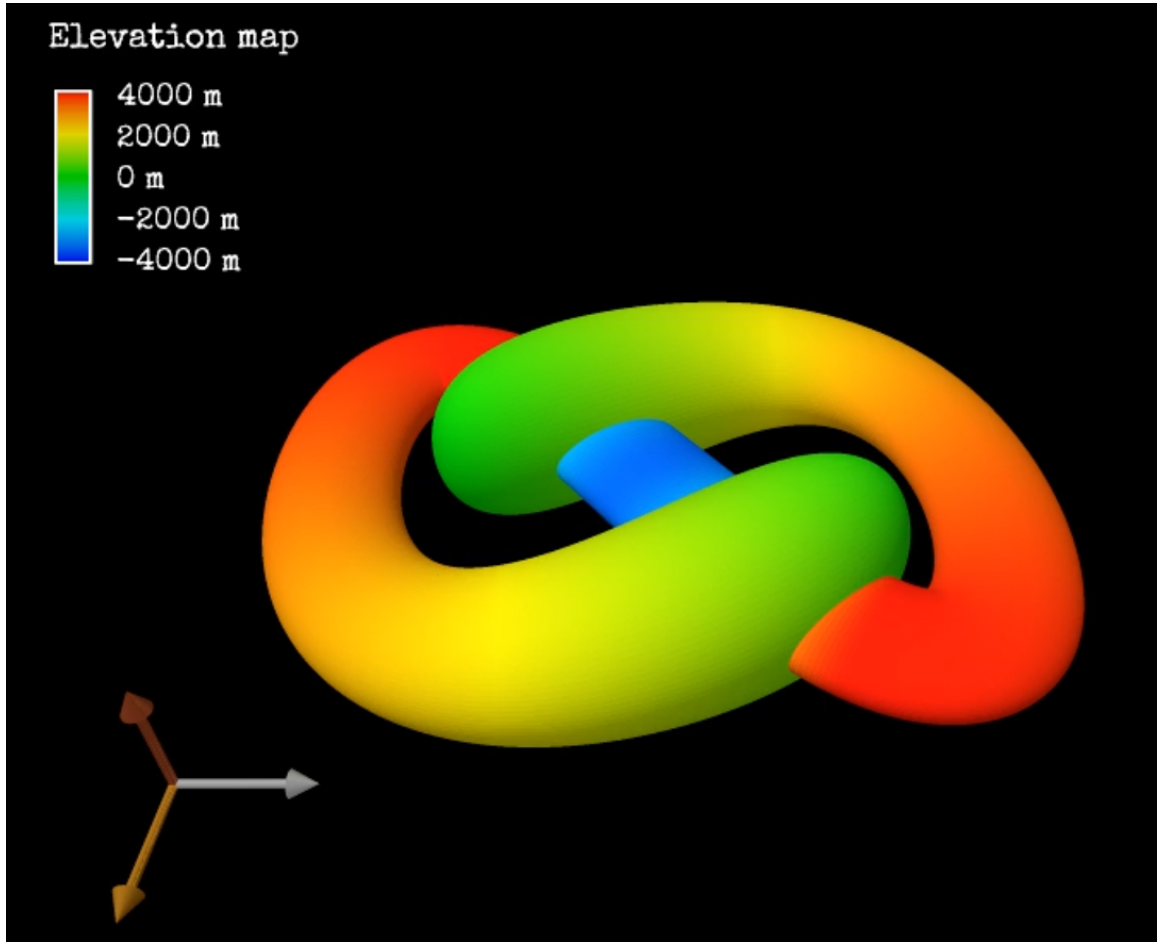
A knotted surface is a submanifold of $\mathbb{R}^4 \cong$ to a closed connected surface

- This works for all surfaces
- There are analogs of Reidemeister moves, *e.g.*:



- The theory of knotted surfaces is a fairly open field of math

The colorful way to 4d



- Color can also be used as the fourth dimension
- What you see is a knotted sphere
- The sphere does not self-intersect

Thank you for your attention!

I hope that was of some help.