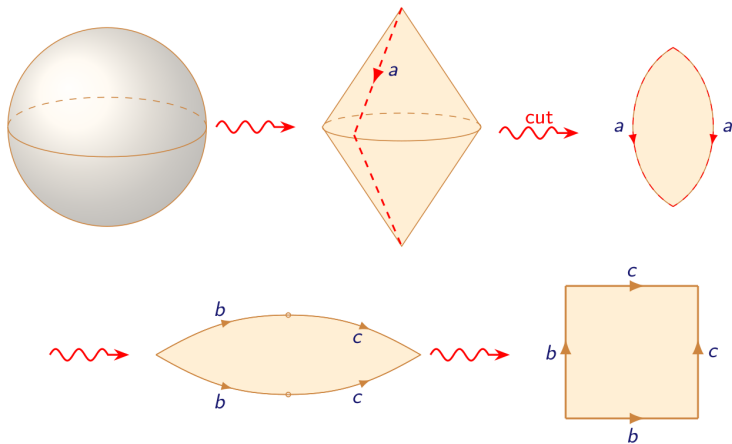


**What are...combinatorial surfaces?**

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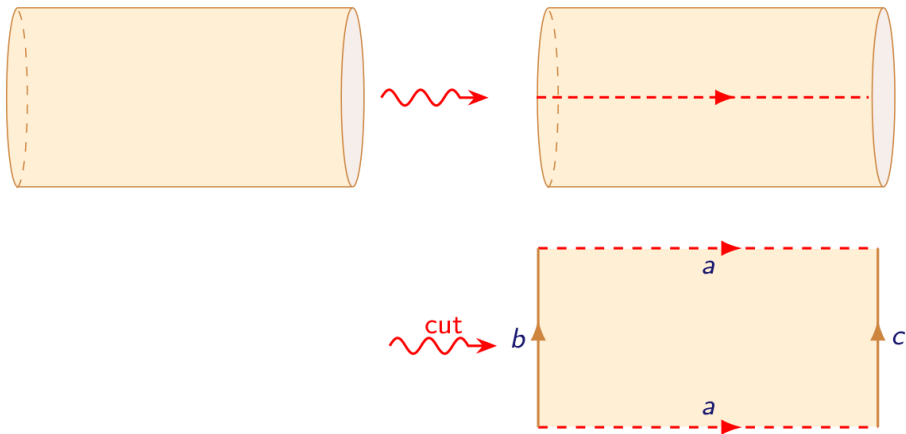
Or: Words and friends

## From a sphere to a rectangle



- ▶ The above **encodes** a sphere as a rectangle with identified sides
- ▶ The process is completely determined by the **words**  $aa^{-1}$  or  $bcc^{-1}b^{-1}$
- ▶ All edges are paired  $\Rightarrow$  **no boundary**

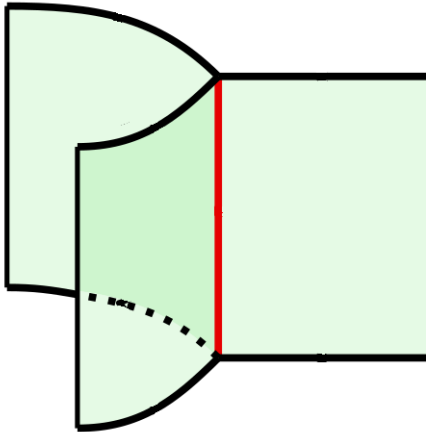
## A cylinder is a rectangle



- ▶ The above **encodes** a cylinder as a rectangle with identified sides
- ▶ The process is completely determined by the **word**  $bac^{-1}a^{-1}$
- ▶ Not paired edges give the **boundary**

## The book with three pages should not appear

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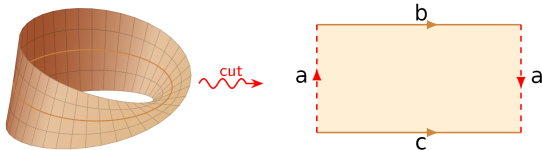


- 
- ▶ The book with three pages is not a surface
  - ▶ Hence we need to avoid identifying **three or more** edges
  - ▶ So that single edges = half discs, paired edges = discs

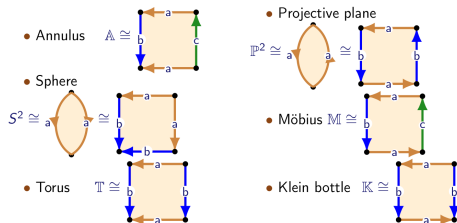
## For completeness: A formal definition

A **combinatorial surface** is a word in  $\{a^{\pm 1}, b^{\pm 1}, \dots\}$  such that every symbol appears at most twice

- Single letters  $\iff$  boundary,  $a^{\pm 1} + a^{\mp 1} \iff$  disc,  $a^{\pm 1} + a^{\pm 1} \iff$  Möbius bands

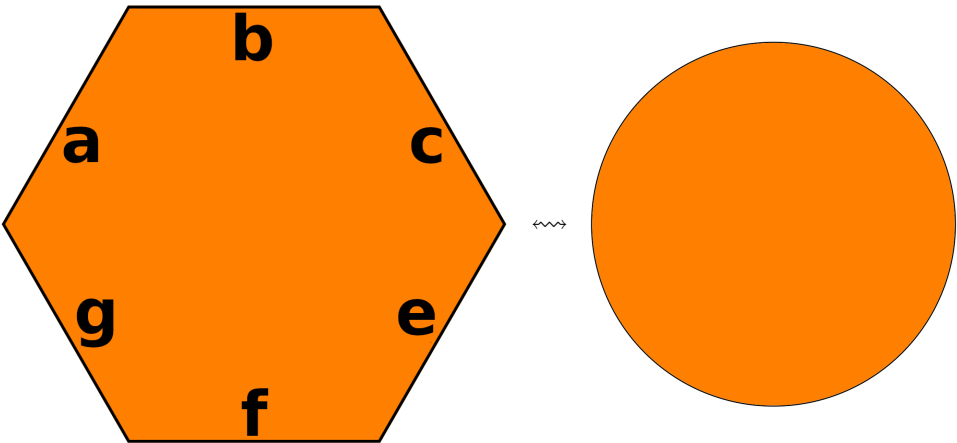


- More examples:



## Relations among words

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- ▶ Words of the form  $abc\dots$  all give discs
  - ▶ Hence, we need to impose **relations** on the words
  - ▶ **Aim** Surfaces  $\leftrightarrow$  words + relations

**Thank you for your attention!**

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I hope that was of some help.