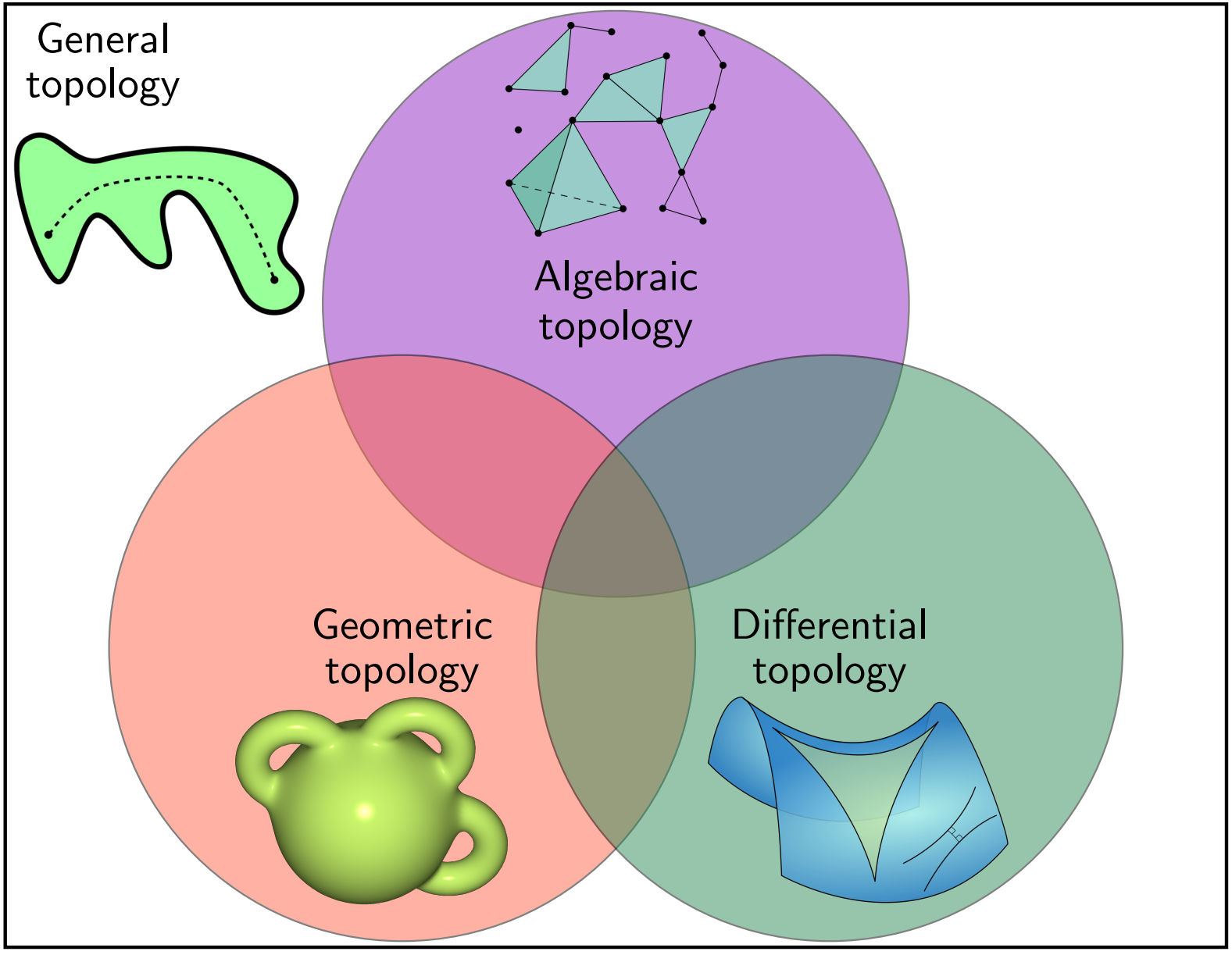
What is...geometric topology?

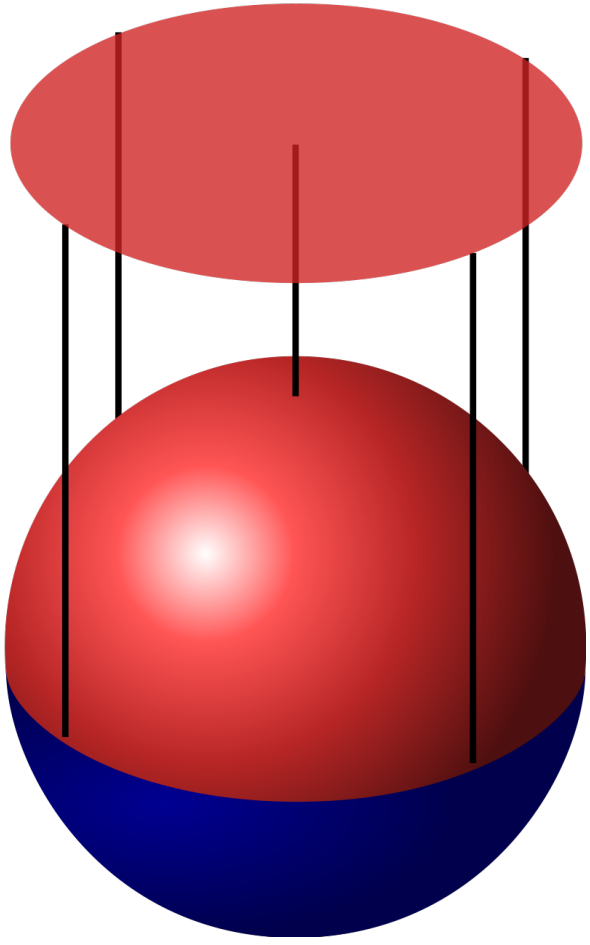
Or: It's a disc!

The main fields of topology



Geometric topology studies objects that are locally discs

Locally a disc

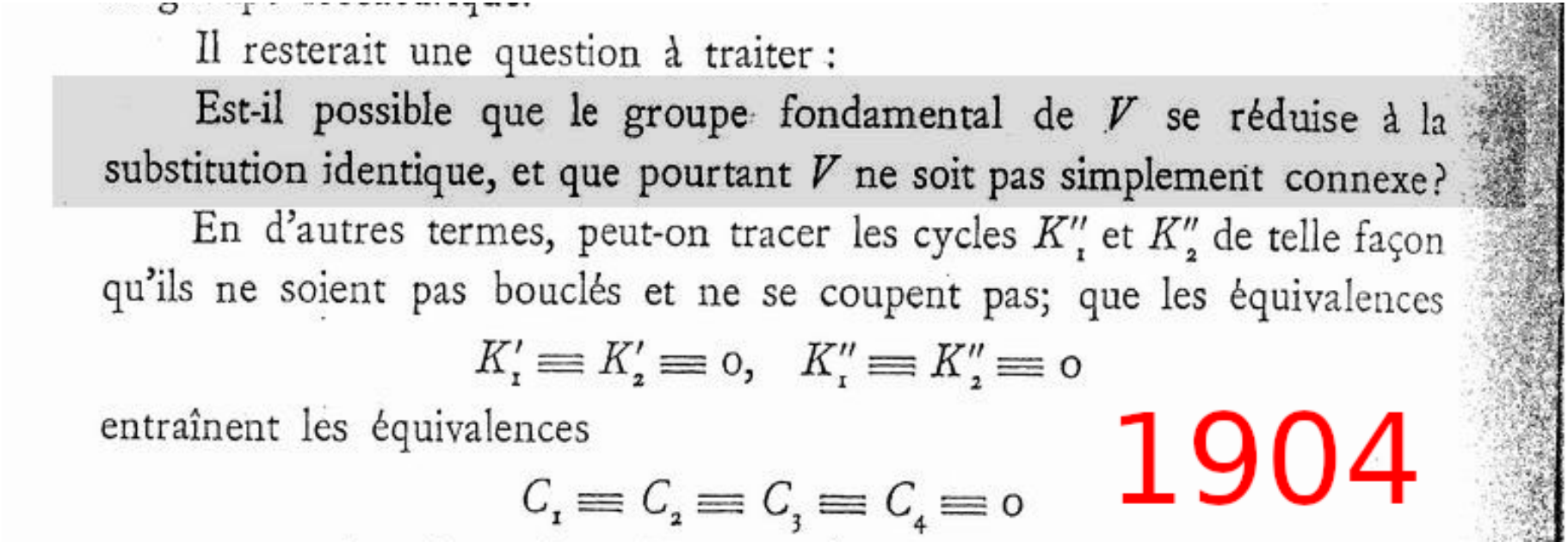


- Manifold (mfd) = discs glued together: sphere, torus, pair of pants, ...
- Geometric topology is the study of manifolds + friends
- This is done without taking area, angles *etc.* into account

The keywords – what geometric topology (for example) studies

- Manifolds extrinsically
  - Knot theory
  - Graphs on surfaces
  - Embeddings
  - ...
- Manifolds intrinsically
  - Construction and classification of 0, 1, 2, ... mfds
  - Heegaard splittings, Kirby calculus, Dehn surgery
  - Poincaré conjecture
  - ...
- More mfds
  - Cobordisms and TQFTs
  - Mapping class groups
  - Unknotting problem
  - ...

Direction one – the Poincaré conjecture



Il resterait une question à traiter :

Est-il possible que le groupe fondamental de V se réduise à la substitution identique, et que pourtant V ne soit pas simplement connexe?

En d'autres termes, peut-on tracer les cycles K''_1 et K''_2 de telle façon qu'ils ne soient pas bouclés et ne se coupent pas; que les équivalences

$$K'_1 \equiv K'_2 \equiv 0, \quad K''_1 \equiv K''_2 \equiv 0$$

entraînent les équivalences

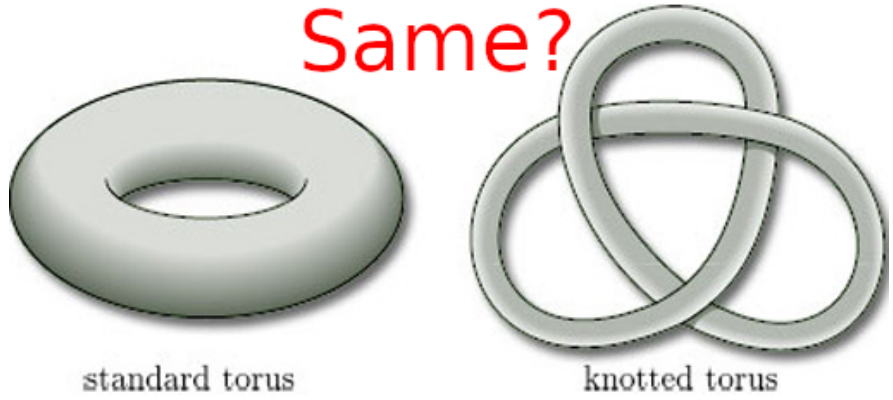
$$C_1 \equiv C_2 \equiv C_3 \equiv C_4 \equiv 0$$

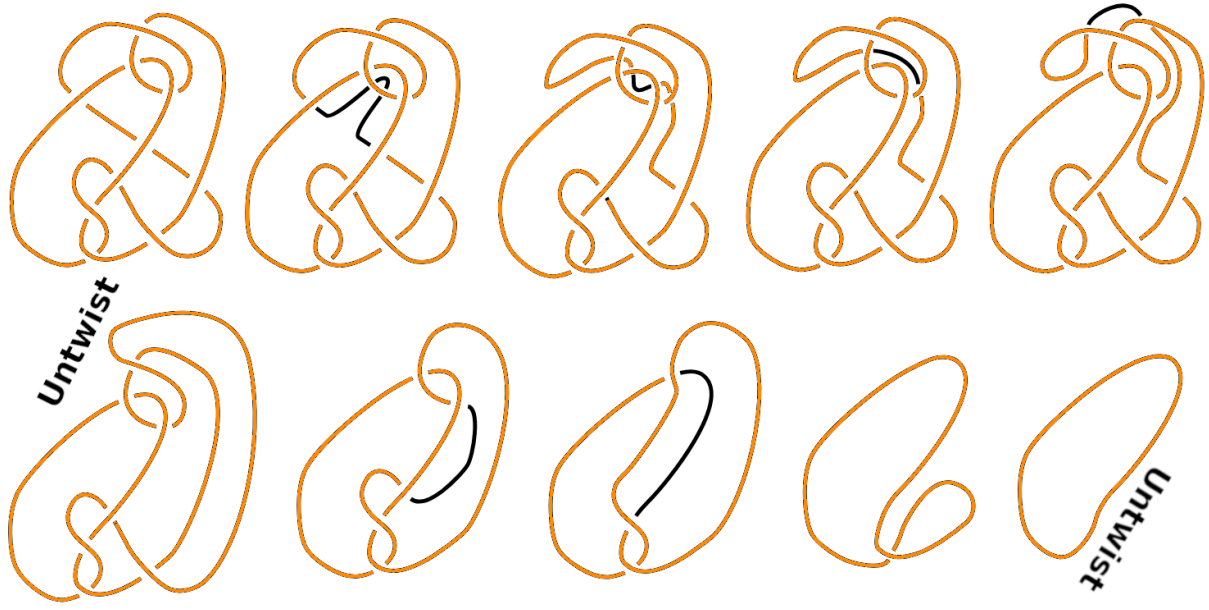
1904

Consider a compact 3-dimensional manifold V without boundary. Is it possible that the fundamental group of V could be trivial, even though V is not homeomorphic to the 3-dimensional sphere?

- A mfd is the sphere $\Leftrightarrow$ a certain algebraic condition is satisfied
- Depending on the interpretation of Poincaré, this is hardest in 3d or 4d
- 3d solved (Perelman & co), 4d widely open (in 2022)

Direction two – unknotting problem





- Knot theory studies embedded mfds up to a different notion than homeomorphism
- One of the main open problems in knot theory: detect the unknot

Thank you for your attention!

I hope that was of some help.