What is...the skeleton?

Or: The bare minimum



- ▶ It is not always preferable to have a lot of data
- ► It is sometimes a good strategy to ignore finer structures
- Skeletons are part of this philosophy

Too big



► KfdVECT Objects finite-dimensional K-vector spaces, arrows K-linear maps

▶ $X, Y \in \mathbb{K}$ fdVECT are isomorphic if and only if they have the same dimension

Good size numbers $\left(\begin{array}{c} a & b & c \\ d & e & f \end{array}\right)$ *n-m* matrix $\mathbb{K}\mathsf{MAT}$

 $\blacktriangleright \quad \mathbb{K}\mathsf{MAT} \quad \mathsf{Objects} \ \mathbb{N}, \ \mathsf{arrows} \ \mathbb{K}\text{-valued matrices}$

► $X, Y \in \mathbb{K}$ MAT are isomorphic if and only if X = Y

- ► A category *S* is skeletal if no two distinct objects are isomorphic
- ▶ A skeleton of C is a skeletal category S equivalent to $C \simeq S$
- ► Theorem Skeletons exists (requires AoC) and are unique up to isomorphism
- ▶ Thus, we can say that *S* is the skeleton of *C* (non-canonical)
- \blacktriangleright Two categories are equivalent \Leftrightarrow they have isomorphic skeletons

Example

- ► KfdVECT is not skeletal
- ▶ KMAT is skeletal
- $\mathbb{K}MAT$ is the skeleton of $\mathbb{K}fdVECT$
- \blacktriangleright An equivalence $\mathbb{K}\mathsf{fd}\mathsf{VECT} \to \mathbb{K}\mathsf{MAT}$ is sending

X to its dimension, f to its associated matrix

Useful or not?



The skeletal form of a category is both always useful and never useful
Roughly If the skeleton is nice, great, if its artificial, probably ignore it

Thank you for your attention!

I hope that was of some help.