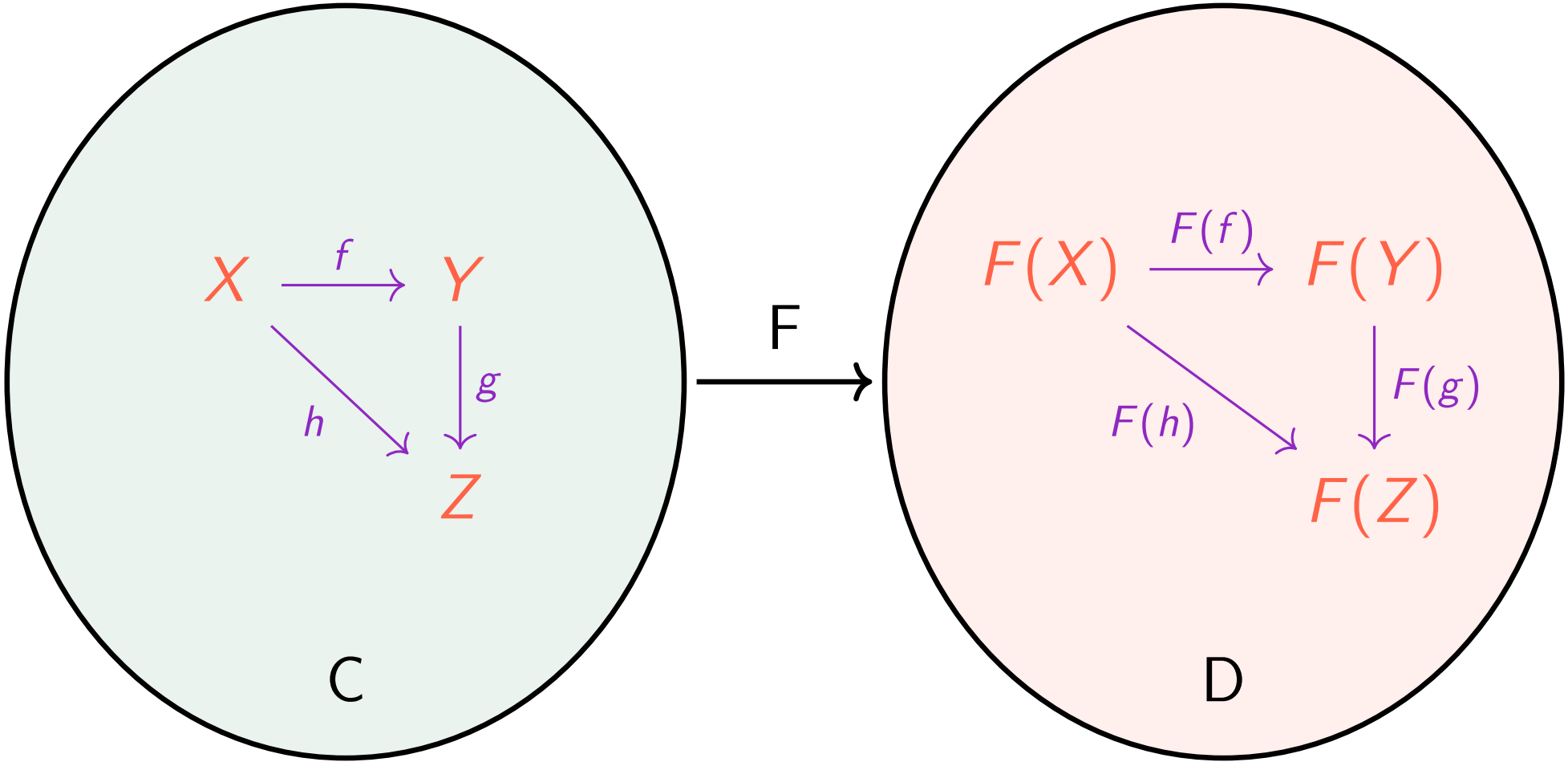
What are...properties of functors?

Or: Preserving diagrams

Functors preserve diagrams



- Functor F: C → D
- F preserves commutative diagrams
- This is crucial and the main point of functors

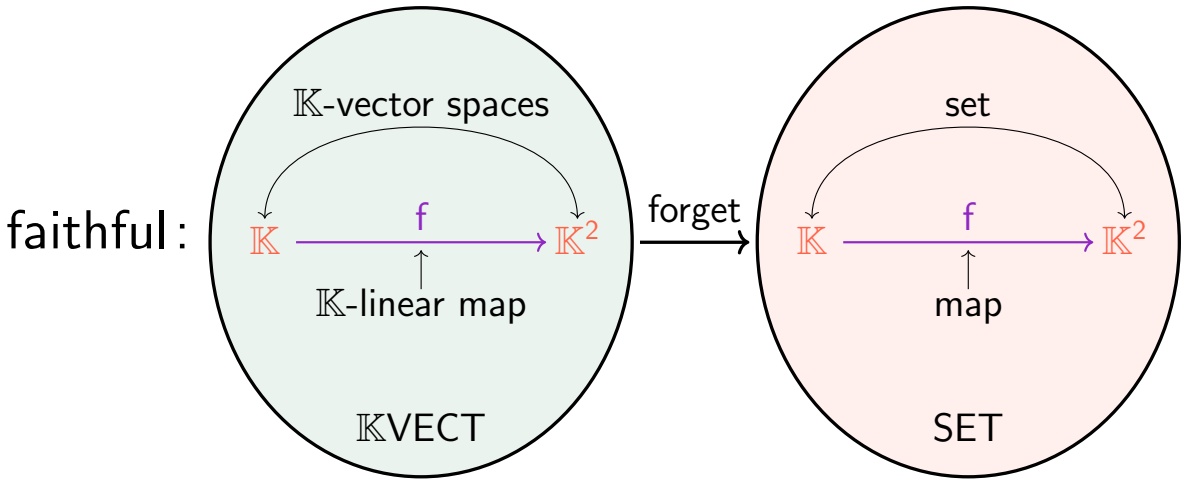
Injective

$$Z \underset{g_2}{\overset{g_1}{\rightrightarrows}} X \xrightarrow{f} Y$$

- A functor F is faithful if

$$F\colon \hom_C(X,Y) \xrightarrow{f \mapsto F(f)} \hom_{F(C)}(F(X),F(Y)) \text{ is injective}$$

- Faithful $\Leftarrow$ monic in CAT but $\not\Rightarrow$ $\Rightarrow$ also needs injective on objects
- Example Forget: $\mathbb{K}$VECT $\to$ SET is faithful



Surjective

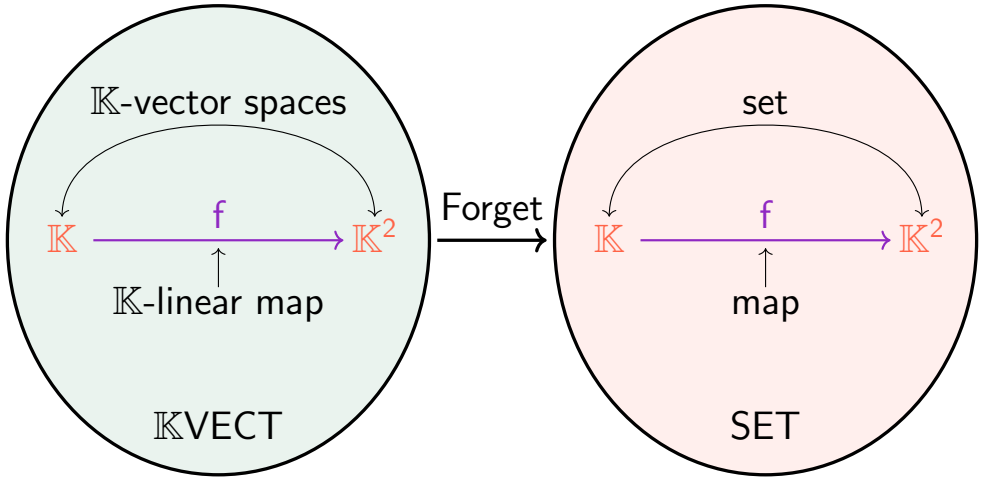
$$X \xrightarrow{f} Y \underset{g_2}{\overset{g_1}{\rightrightarrows}} Z$$

- A functor F is full if

$$F\colon \hom_C(X,Y) \xrightarrow{f \mapsto F(f)} \hom_{F(C)}(F(X),F(Y)) \text{ is surjective}$$

- Full $\Rightarrow$ epic in CAT but $\not\Leftarrow$ I do not know any good characterization of $\Leftarrow$
- Example Forget: $\mathbb{K}$VECT $\to$ SET is not full

not full:



For completeness: A list of properties

Any functor F satisfies:

- F preserves commutative diagrams
- F preserves identities ($f = id \Rightarrow F(f) = id$)
- F preserves isomorphisms (f iso $\Rightarrow F(f)$ iso)

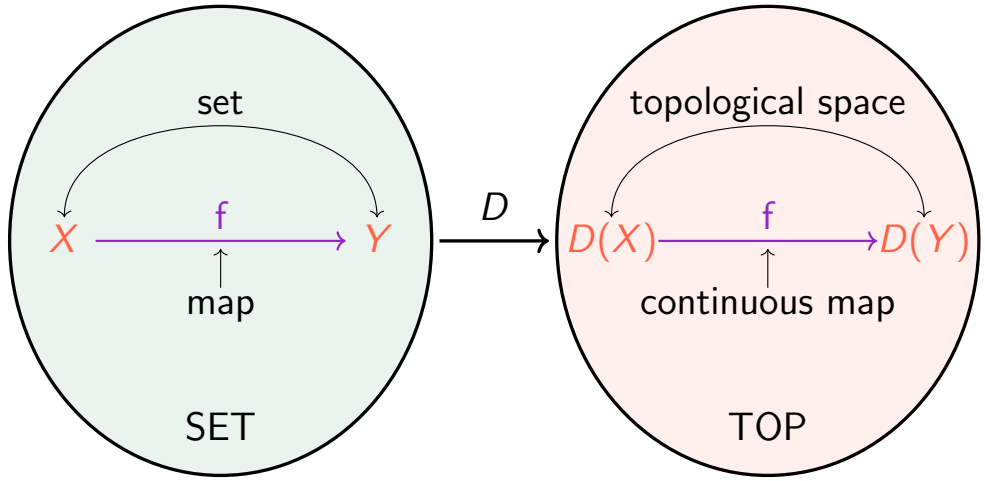
Some functors F satisfy:

- F is faithful
- F is an embedding (= faithful and injective on objects)
- F is full
- F is iso (= fully faithful and bijective on objects)
- F reflects identities ($F(f) = id \Rightarrow f = id$)
- F reflects isomorphisms ($F(f)$ iso $\Rightarrow f$ iso)
- Many more that we will meet

Examples of functors

- Forget: $\mathbb{K}$VECT $\to$ SET is faithful, but not full or an embedding
- Powerset: SET $\to$ SET is an embedding, but not full
- Demetrization: $\mathrm{MET}_c \to$ TOP is fully faithful, but not an embedding
- Discrete topology D: SET $\to$ TOP is a full embedding

full embedding:



- Indiscrete topology I: SET $\to$ TOP is a full embedding
- Inclusion functor: $\mathbb{Z}$MOD $\to$ GROUP is a full embedding
- Many more that we will meet

Thank you for your attention!

I hope that was of some help.