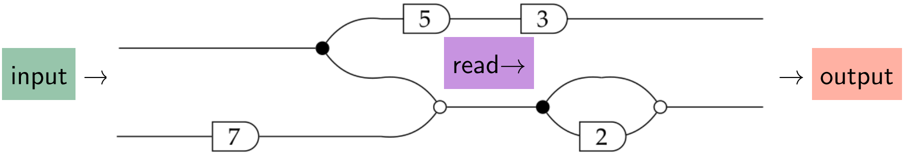
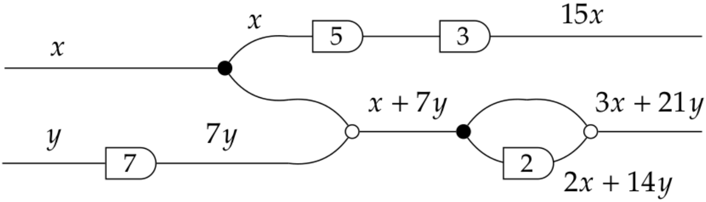
What are...props?

Or: Applications 2 (category theory in signal processing)

Signal flow graphs (SFG); simplified





SFGs have three types of signal processing units:

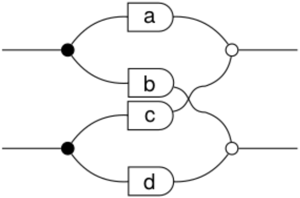
- Coupons multiply inputs by a given number
- Black splits take inputs and produce two copies
- White merges take inputs and produce the sum

SFG and matrices

building blocks:

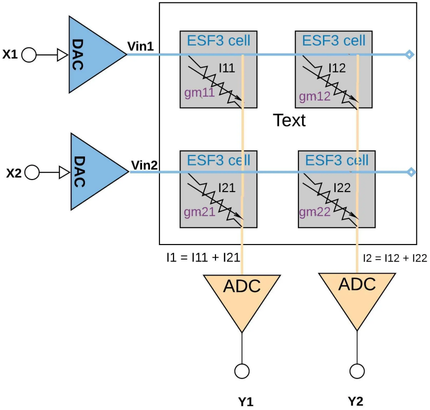
generator	icon	matrix	arity
amplify by $a \in R$		(a)	$1 \to 1$
add		$\begin{pmatrix}1\\1\end{pmatrix}$	$2 \to 1$
zero		$()$	$0 \to 1$
copy		$\begin{pmatrix}1 & 1\end{pmatrix}$	$1 \to 2$
discard		$()$	$1 \to 0$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \leftrightsquigarrow$



- Matrices can be modeled using SFGs
- Actually, the whole symmetric monoidal category $\mathbb{K}$MAT can be modeled
- In particular, we get matrix multiplication

Matrix multiplication is everywhere



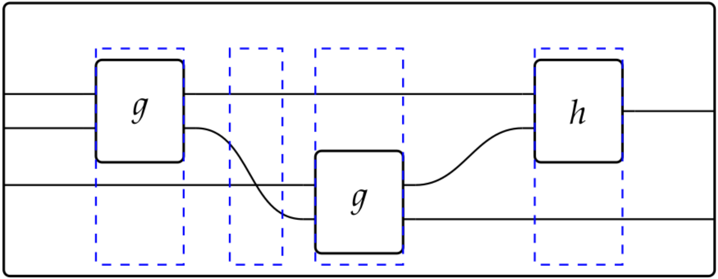
- The diagrammatic matrix multiplication of SFG is very applicable
- Examples are analog computers that are used in machine learning
- Here each building block corresponds to some "analog chip"

For completeness: A formal definition

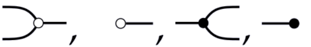
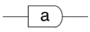
A prop is a symmetric monoidal category on objects $\mathbb{N}$ with $\otimes = +$ on objects

- Props are denoted using string diagrams (traditionally read sideways)

$$(g \otimes id_1) \circ (id_1 \otimes \sigma) \circ (id_1 \otimes g) \circ (h \otimes id_1) \rightsquigarrow$$



- SFGs are props with arrow generators

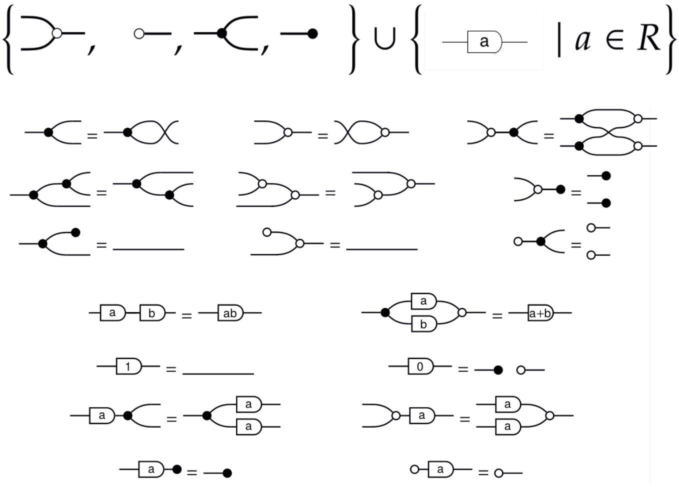
$\Big\{$  $\Big\} \cup \Big\{$ 

$\mid a \in R \Big\}$

A presentation for $\mathbb{K}$fdVECT

gens:

rels:



- The above is a symmetric monoidal generator-relation presentation of $\mathbb{K}$MAT
- Since $\mathbb{K}$MAT is the skeleton of $\mathbb{K}$fdVECT, we get the same for the latter
- This, appropriately formulated, works over any ring R

Thank you for your attention!

I hope that was of some help.