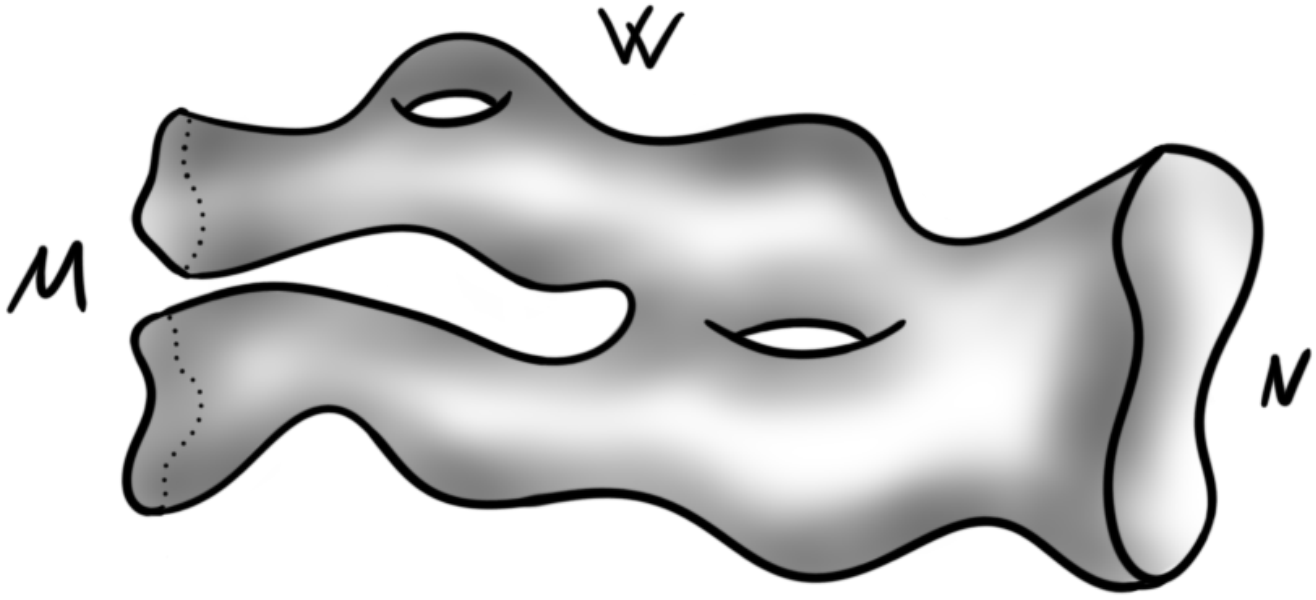
What are...TQFTs?

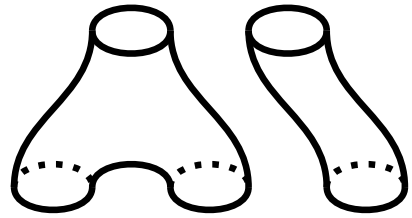
Or: Applications 1 (category theory in mathematics)

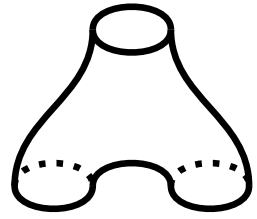
A topological version of "map"



- $(n+1)$-cobordism is a $(n+1)$dim manifold W with $n$dim boundary M, N
- With appropriate care one can talk about source and target of cobordisms
- We can see them as maps between M and N
- 2D This video will stay with $n = 1$ because I like to draw pictures ;-)

The category 2COB

$f =$ 

$g =$ 

$\bigcirc$

$g \uparrow$

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$f \uparrow$

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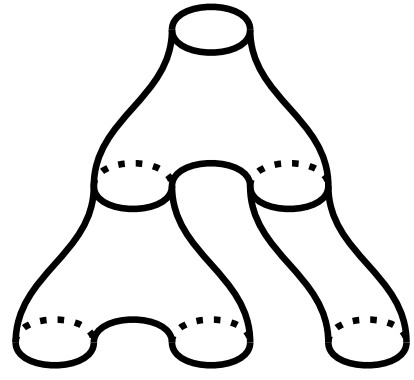
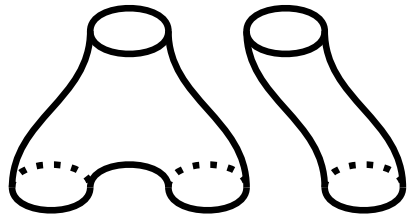
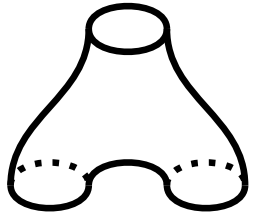
$g \circ f =$ 

- Objects of 2COB are 1-manifolds Circles
- Arrows of 2COB are (1+1)-cobordisms Surfaces
- I draw them bottom to top

The symmetric monoidal category 2COB

$f =$ 

$g =$ 

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$g \otimes f \uparrow$

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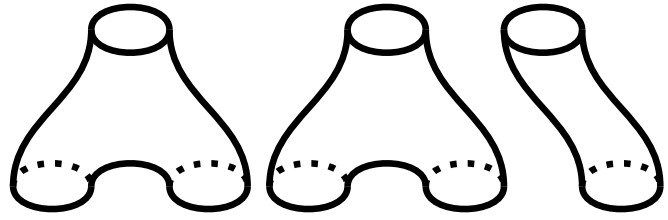
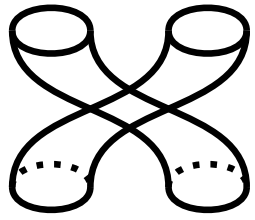
$g \otimes f =$ 

- 2COB is monoidal with $\otimes$ being "juxtaposition"
- 2COB is symmetric with symmetry being

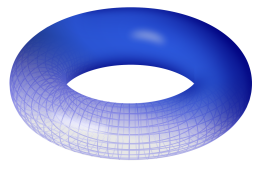


For completeness: A formal definition

A TQFT Z is a symmetric monoidal functor

$$Z\colon \mathrm{2COB} \to \mathbb{K}\mathrm{fdVECT}$$

- There are more general definitions that works in higher dimension
- TQFTs play an important role in topology because

$Z\left(\right.$  $\left.\right)$ is an invariant

Works for all closed (n+1)-manifolds

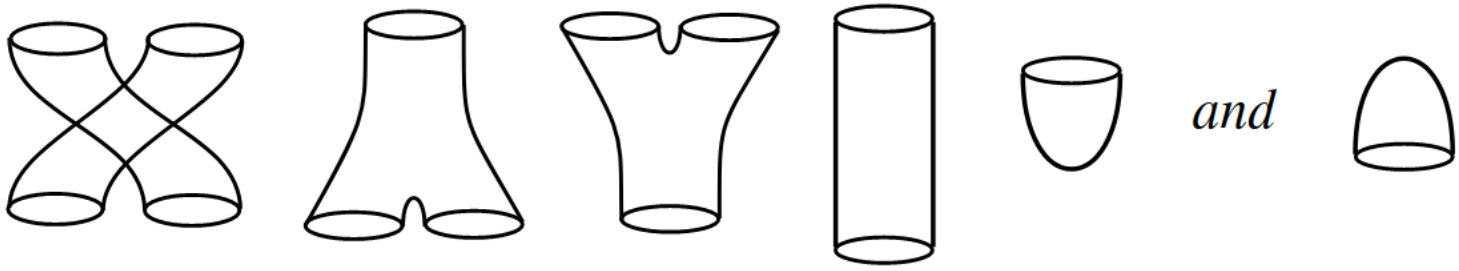
- TQFTs actually arose in physics
- 2D TQFTs are completely classified :

Equivalence of categories: $\mathrm{2DTQFT} \cong \mathrm{cFROB}$

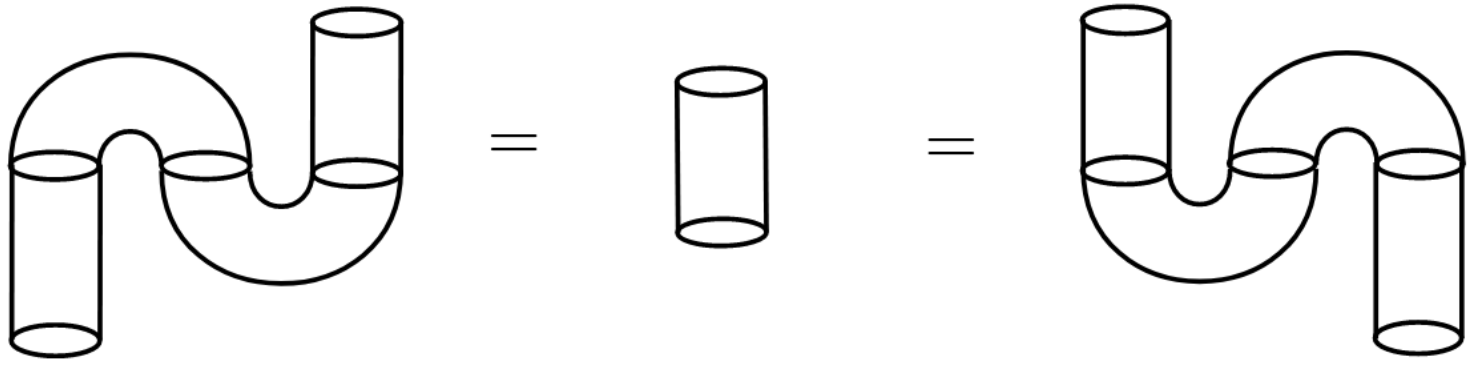
where cFROB are commutative Frobenius algebras

A presentation for 2COB

gens:



rels (example):



- The above is a symmetric monoidal generator-relation presentation of 2COB
- Ok, not quite: there are more relations and I was lazy ;-)
- The relation list (very finite!) can be found *e.g.* in Kock's book *Frobenius Algebras and 2-D Topological Quantum Field Theories*

Thank you for your attention!

I hope that was of some help.