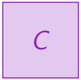
What are...string diagrams, take 2?

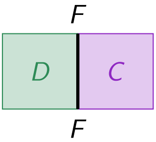
Or: Two-dimensional algebra

String diagrams – reminder

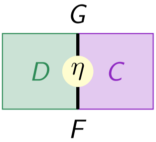
- 2-cell Draw a category C as a face



- 1-cell Draw a functor $F\colon C \to D$ as a line



- 0-cell Draw a nat trafo $\eta\colon F \Rightarrow G$ as a point



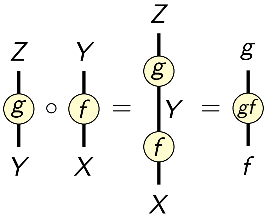
String diagrams for monoidal categories

- No face colors needed!
- 1-cell Draw an object $X \in C$ as a line

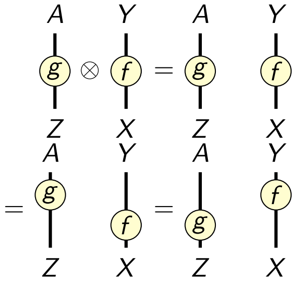
- 0-cell Draw an arrow $f \colon X \to Y$ as a point

Composition

- The usual/categorical composition $\circ$ is vertical stacking



- The monoidal product $\otimes$ is horizontal stacking

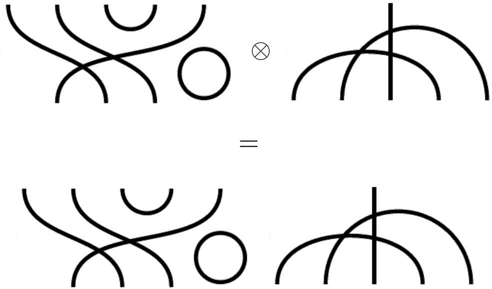


For completeness: A formal definition

String diagrams for monoidal categories have...

- ...?? represented by a portion of plane, We will see later what ?? is
- ...objects represented by strings 1d
- ...arrows by coupons 0d
- ..."evident" composition rules

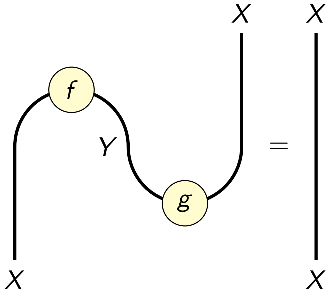
The prototypical monoidal category is thus 1COB (in some sense):



Something is missing, right?

Theorem. Two diagrams are equivalent if they are related by scaling or by a planar isotopy keeping the upwards orientation

missing :



- This is not quite a planar calculus – not always
- This gap is filled by quantum algebra
- Keywords Rigid and pivotal categories

Thank you for your attention!

I hope that was of some help.