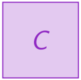
# What are...string diagrams?

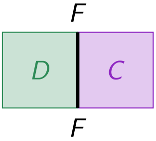
Or: Diagrammatic algebra

## String diagrams

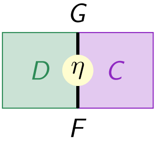
- **2-cell** Draw a category $C$ as a face




- **1-cell** Draw a functor $F\colon C \to D$ as a line

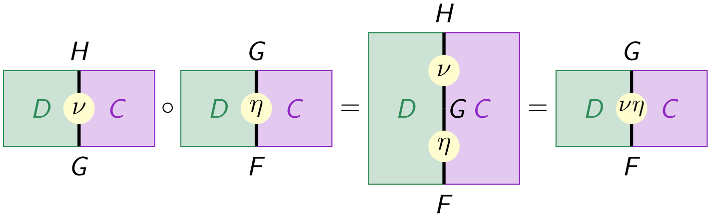



- **0-cell** Draw a nat trafo $\eta\colon F \Rightarrow G$ as a point

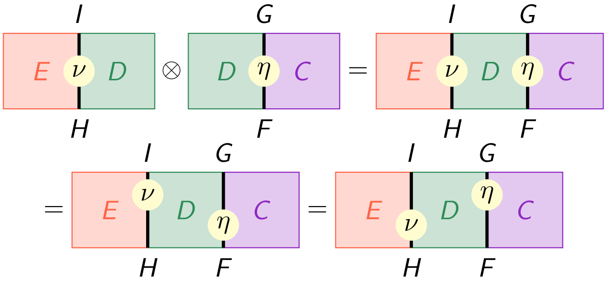


## Composition

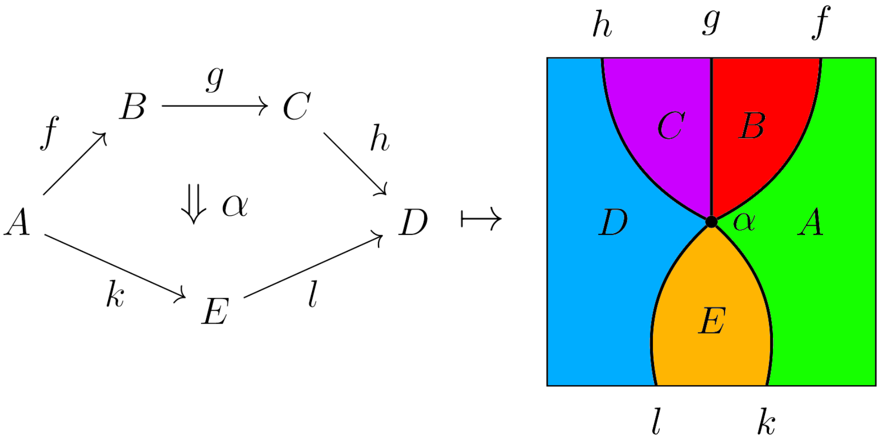
- $\nu\eta = \nu \circ \eta$ for $\eta \colon F \Rightarrow G$, $\nu \colon G \Rightarrow H$ is vertical stacking




- $\nu \otimes \eta$ for $\eta \colon F \Rightarrow G$, $\nu \colon H \Rightarrow I$ is horizontal stacking



## Poincaré duality




- Using classical diagrams we have 0,1,2 (objects, arrows, nat trafos)
- In string diagrams its 2-1-0
- String diagrams and classical diagrams are Poincaré dual

## For completeness: A formal definition

String diagrams have...

- ...objects represented by a portion of plane, 2d
- ...arrows represented by strings 1d
- ...nat trafos by coupons 0d
- ..."evident" composition rules

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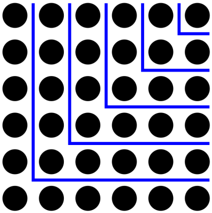
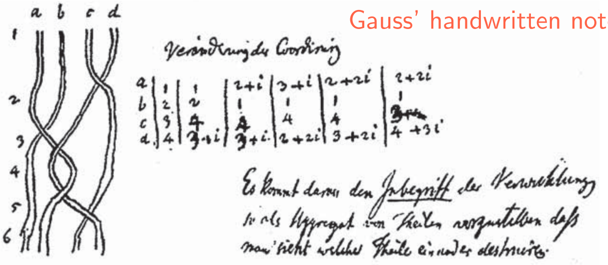
- String diagrams as part of diagrammatic algebra
- The aim is to formalize and mechanizing diagrammatic reasoning
- The point is that diagrammatic proofs (broadly interpreted) are powerful



$$n^2 = 1 + 3 + 5 + \cdots + (2n - 1)$$

# Diagrammatics



Gauss' handwritten notes

Diagrammatics has been around for Donkey's years, *e.g.*:

- Gauss ∼1800 (hard to gauge) Electromagnetism
- Rumer–Teller–Weyl ∼1932 Quantum chemistry
- Brauer ∼1937 Representation theory; 1COB is stolen from Brauer
- Feynman ∼1948 Subatomic particles
- Penrose ∼1972 Tensor calculus

**Thank you for your attention!**

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I hope that was of some help.