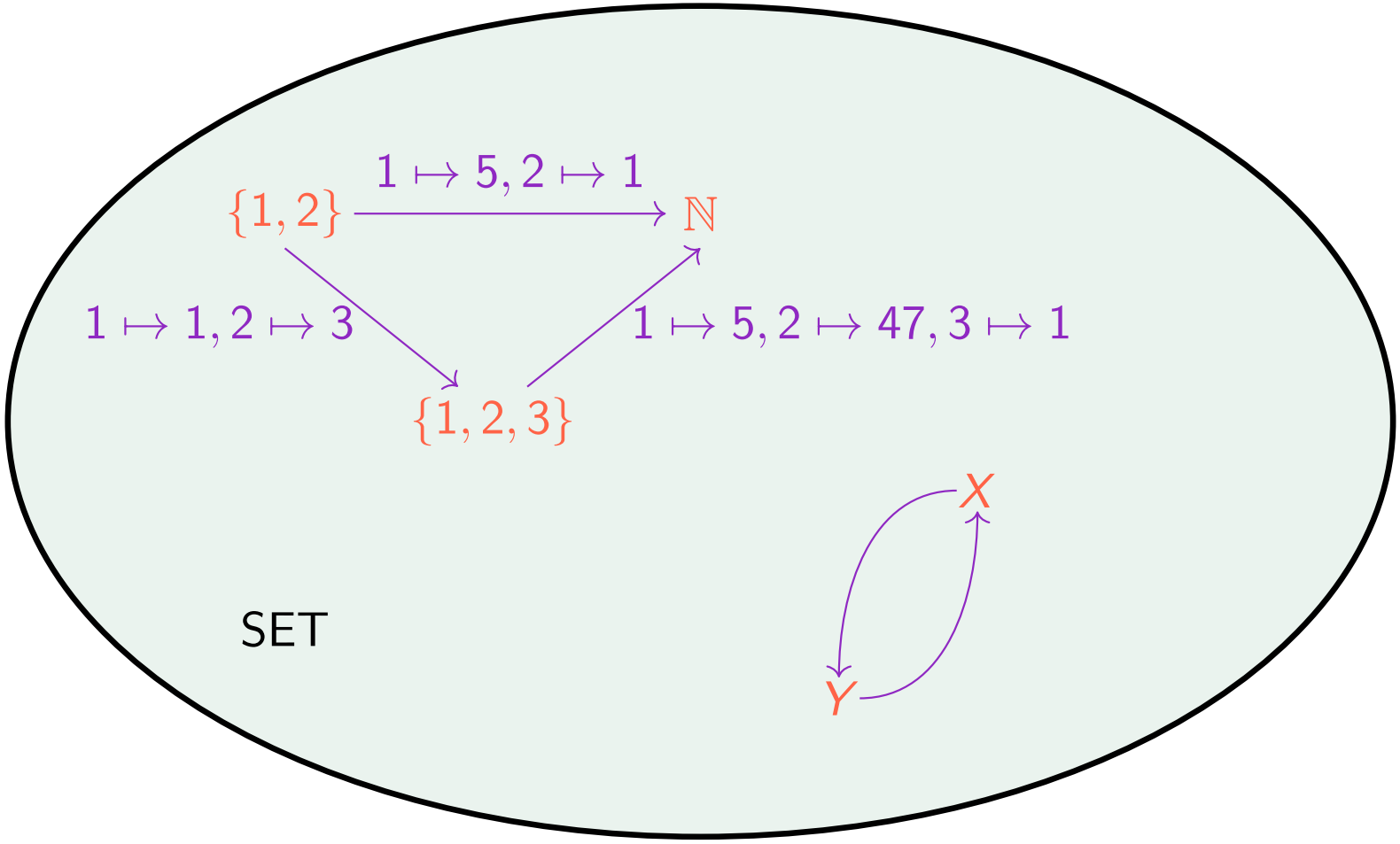
What is...a category?

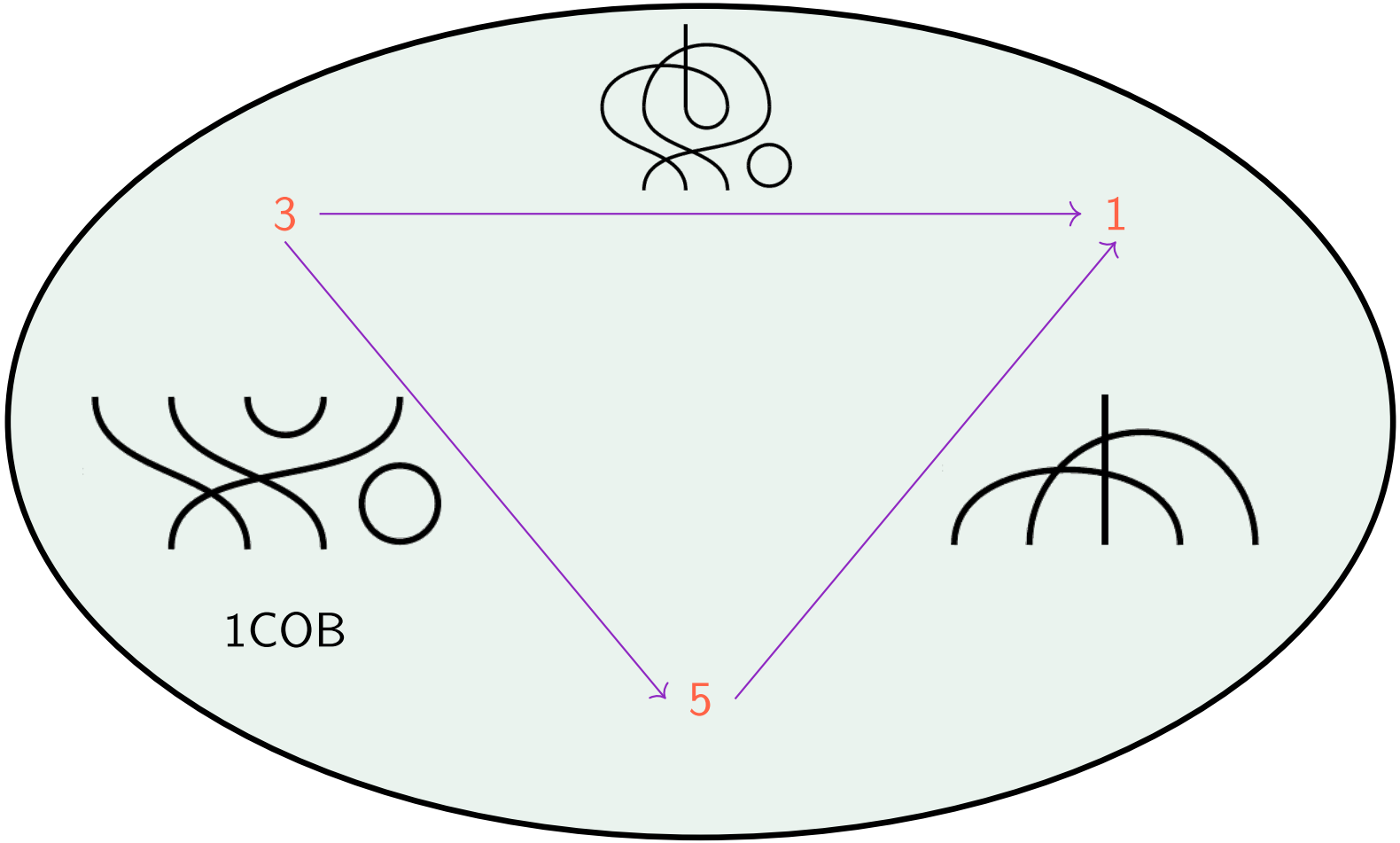
Or: Arrows in action

Collections of sets



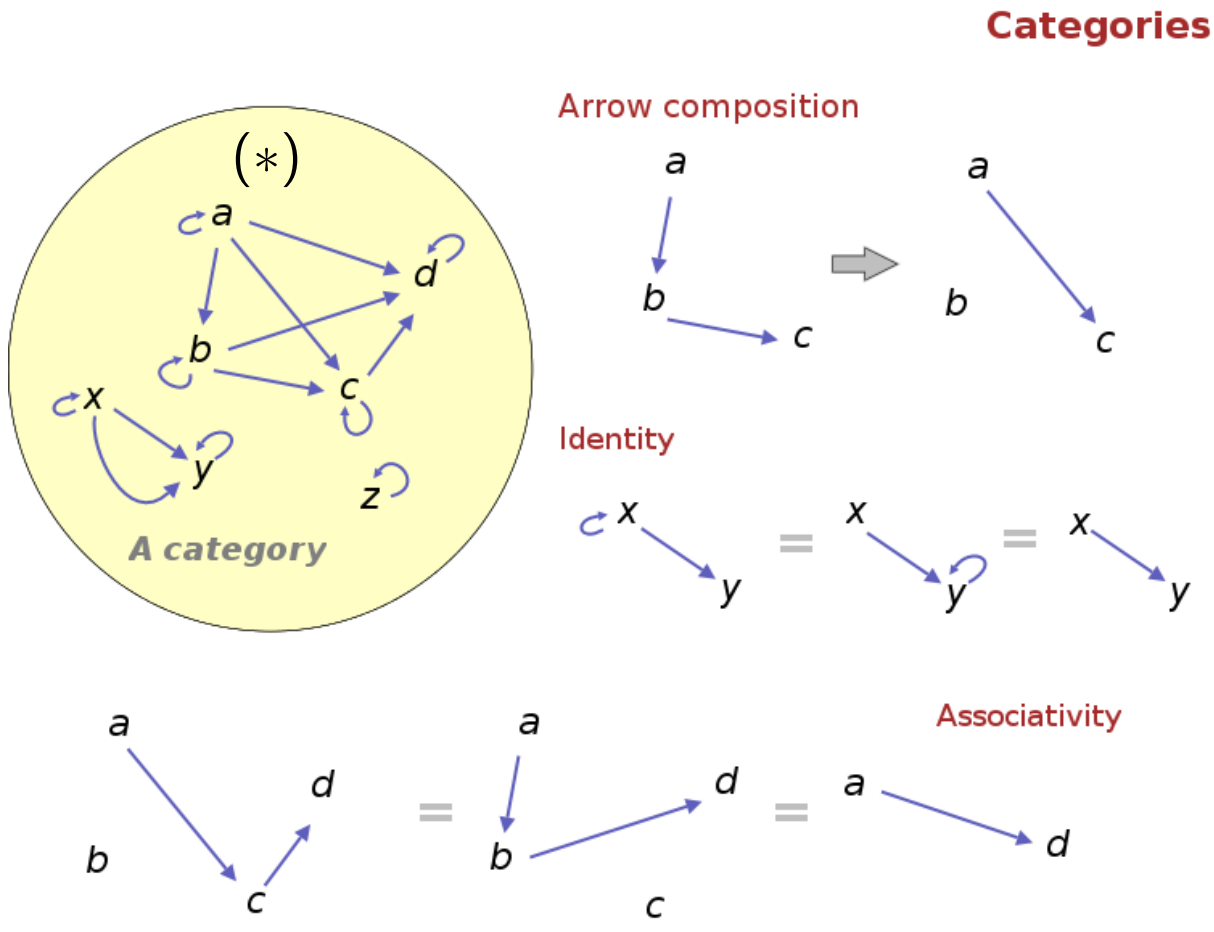
- The collection of sets SET "Category" contains sets "Objects"
- It also contains maps "Arrows"
- We can compose maps in a nice way

Collections of 1-manifolds



- The collection 1COB "Category" contains points (0-manifolds) "Objects"
- It also contains lines (1-manifolds) "Arrows"
- We can compose lines in a nice way

Arrows et al. – the similarities



- SET and 1COB have objects and arrows
- SET and 1COB have composition
- SET and 1COB have identities (I will often ignored these)
- SET and 1COB satisfy associativity

For completeness: A formal definition

A category C is a quadruple $C = (Ob(C), \hom_C, id, \circ)$ consisting of:

- A class $Ob(C)$ of object
- For $X, Y \in Ob(C)$ a set $\hom_C(X, Y)$ of arrows
- For $X \in Ob(C)$ and identity arrow id_X
- A composition for $f : X \to Y$ and $g : Y \to Z$ denoted $gf = g \circ f : X \to Z$

such that:

- $\circ$ is associative
- id_X are identities
- the sets $\hom_C(X, Y)$ are pairwise disjoint

"Like a set with arrows"
"Like a group with multiple start points"
"Like a universe where relations=arrows matter"

Some examples

Name	Objects	Arrows	Concrete?
SET	Sets	Maps	Yes
1COB	0-manifolds	1-manifolds	No
nCOB	(n-1)-manifolds	n-manifolds	No
fSET	Finite sets	Maps	Yes
pSET	Sets	Partial maps	Yes
GROUP	Groups	Group homomorphisms	Yes
TOP	topological spaces	continuous map	Yes
oTOP	topological spaces	continuous open maps	Yes
$\mathbb{K}$VEC	$\mathbb{K}$-vector spaces	$\mathbb{K}$-linear map	Yes
$\mathbb{K}$MAT	$\mathbb{N}$	$\mathbb{K}$-valued matrices	No
$\bullet$	$\bullet$	$id_\bullet$	No
$(*)$	What you see	What you see	No

- Categories need not to be concrete= set based Arrows$\neq$maps
- Arrows=main players but categories are often named by their objects

Thank you for your attention!

I hope that was of some help.