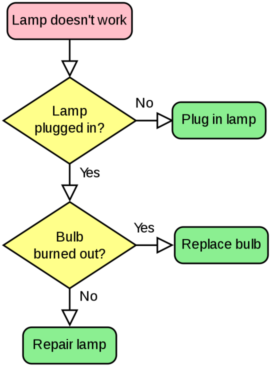
What is...category theory?

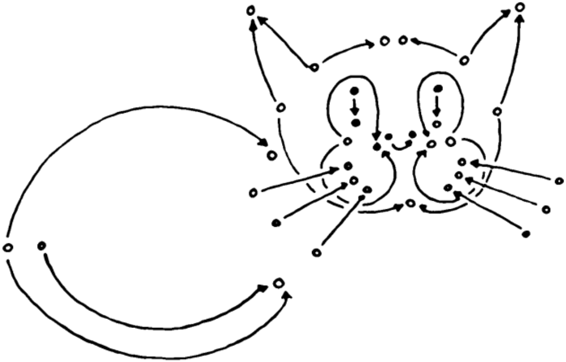
Or: Organizing ideas

Flowcharts=categories!?

Flowcharts (FC)

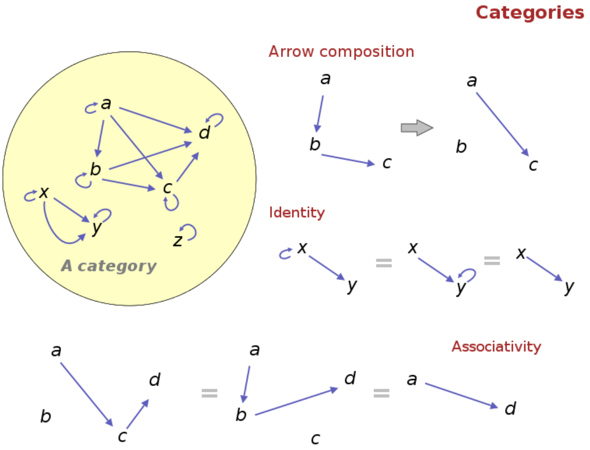


Categories (C)



- Flowcharts A symbolism that allows one to organize complicated facts
- Category theory A symbolism that allows one to organize complicated facts

Category theory could also be called Arrow theory



- In FC and C Arrows $\leftrightsquigarrow$ Relations between things
- SET Vertices=sets, arrows=maps=relations between sets
- Misnomer Categories are usually named after the vertices not their arrows

The keywords – what (a classical course in) category theory studies

- Categories a.k.a. categories
  - Diagram chase
  - Universal properties and limits
  - The Yoneda Lemma
  - ...
- Functors a.k.a. arrows between categories
  - Equivalence
  - Adjoint functors
  - Monads
  - ...
- Natural transformations a.k.a. arrows between arrows between categories
  - 2-categories
  - Kan extensions
  - Graphical calculus
  - ...

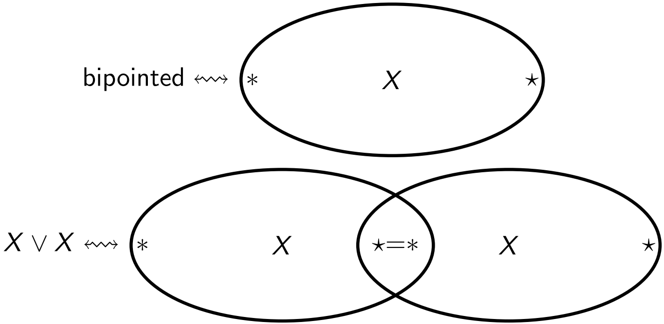
Direction one – universal properties of everyday objects

Question: What makes $[0,1]$ special?

Idea: Find an universal property

Slogan: Universal property = why we always end up with the same concept

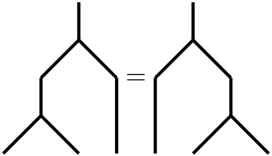
- $[0,1]$ = the terminal bipointed space equipped with a map of bipointed spaces $X \to X \vee X$ ("times 2")
- Informally $[0,1]$ gives path and path composition, and is universal as such



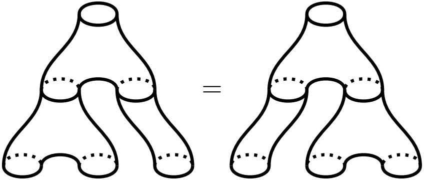
Direction two – graphical language of categories

Associativity: $(ab)c = a(bc)$

String version:



Topological version:



- Diagrammatic notation is useful Relations become visually clear
- Diagrammatic notation is general Generalizations of classical results
- Diagrammatic notation is beautiful Very biased ;-)

Thank you for your attention!

I hope that was of some help.