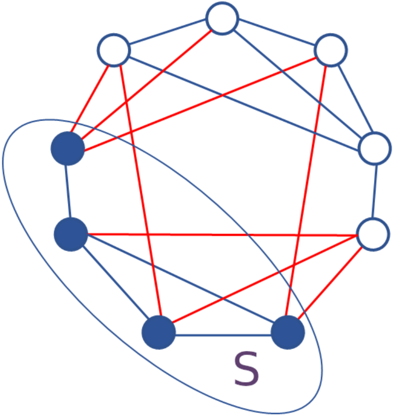
What is...the edge expansion constant?

Or: The second largest - part 2

Cutting into two



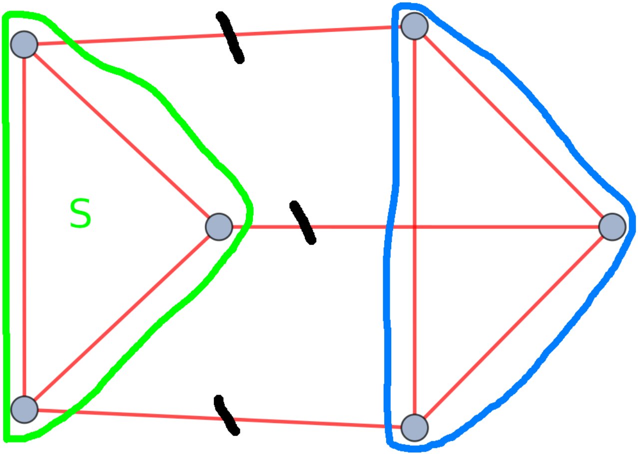
$, \quad |\partial S| = 8$

- Given a subset S of the vertices of G
- Cut edges until S and $G \setminus S$ are disconnected
- Denote that number by ∂S ("boundary of S")

Fair cuts

- Let $h(G) = \min_{S, 0<|S|\le n/2} |\partial S|/|S|$ Edge expansion or Cheeger's constant
- We give $|\partial S|$ the weight $1/|S|$ to make bigger S more attractive
- Task Compute $h(G)$

Large h ⇝ hard to disconnect

$G_3 =$ 

- Take two complete graphs K_n; above $k = 3$
- Connected i vertices one-by-one and get graphs G_0, G_1, G_2, ... , G_n
- Then $h(G_0) = 0$, $h(G_1) = 1/n$, $h(G_2) = 2/n$, ..., $h(G_n) = n/n = 1$

For completeness: A formal statement

For a k-regular graph not K_1, K_2, K_3 we have

$$1/2(k - \lambda_2) \leq h(G) \leq \sqrt{k^2 - \lambda_2^2}$$

Here λ_2 is the second largest eigenvalue

- Expander graphs are:

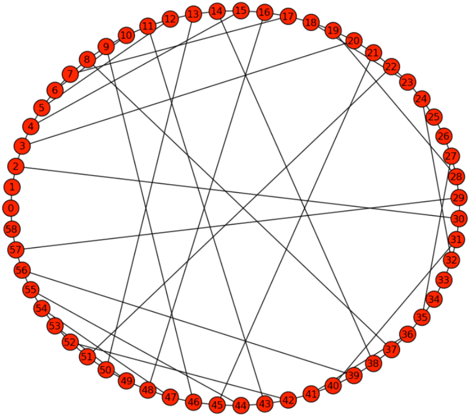
Definition. A sequence of (non-oriented, finite) graphs $(\Gamma_n)_{n\geq 1}$ is a *family of expanding graphs* if

- The number of vertices of Γ_n tends to infinity as n tends to infinity;
- There exists $k \geq 1$ such that the degree of *each* vertex of *each* graph is at most k (the graphs are not too dense);
- There exists $\delta > 0$ such that $h(\Gamma_n) \geq \delta$ for all n (the Cheeger constant is uniformly bounded away from zero).

Such graphs are simultaneously sparse and highly connected.

- In this video this means edge expansion but can also use other definitions
- Expanders became famous because of their role in sorting networks

Expanders



- Not trivial: Do expanders exist?
- Expanders have many applications so we want many examples (more another time)
- Example Vertices $\{0, ..., p(prime) - 1\}$, connect $a \neq 0$ to $a \pm 1$ mod p and a^{-1} mod p, and 0 to $0, 1, p - 1$, gives a family of expanders

Thank you for your attention!

I hope that was of some help.