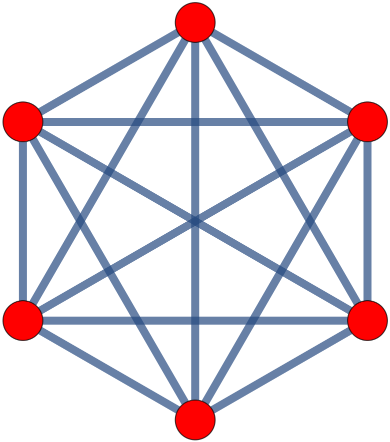
What are...example of spectra?

Or: Let us compute!

Spectrum of complete graphs

$K_6 =$ 

- Complete graph K_n "=" everything is connected to everything
- Spectrum $S(K_n)$ $= \{n-1, (-1)^{n-1}\}$, PF eigenvector $= (1, ..., 1)$
- There are many paths in K_n

Spectrum of path graphs

Path graphs P_n, for $n \geq 1$ (also called line graphs)

Vertex set $V = \{1, 2, \ldots, n\}$

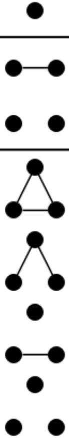
Edge set $E = \{\{1,2\}, \{2,3\}, \ldots, \{n-1, n\}\}$

$P_6 =$ 

$P_7 =$ 

- Path graph P_n "=" a line
- Spectrum $S(P_n)$ $= \{2\cos(k\pi/(n+1)) | k = 1, \ldots, n\}$, PF eigenvector $=$ $(1, [2], [3], \ldots, [3], [2], 1)$ for $[a] = \exp(\pi i/(n+1))^a - \exp(\pi i/(n+1))^{-a} / \exp(\pi i/(n+1))^1 - \exp(\pi i/(n+1))^{-1}$
- There are very few paths in P_n

Spectra of small graphs



1.1	0
2.1	$1, -1$
2.2	$0, 0$
3.1	$2, -1, -1$
3.2	$\sqrt{2}, 0, -\sqrt{2}$
3.3	$1, 0, -1$
3.4	$0, 0, 0$

4.1	$3, -1, -1, -1$
4.2	$\rho, 0, -1, 1-\rho$
4.3	$2, 0, 0, -2$
4.4	$\theta_1, \theta_2, -1, \theta_3$
4.5	$\sqrt{3}, 0, 0, -\sqrt{3}$
4.6	$\tau, \tau-1, 1-\tau, -\tau$
4.7	$2, 0, -1, -1$
4.8	$\sqrt{2}, 0, 0, -\sqrt{2}$
4.9	$1, 1, -1, -1$
4.10	$1, 0, 0, -1$
4.11	$0, 0, 0, 0$

- $\tau = (1+\sqrt{5})/2 \approx 1.62$, and $\rho = (1+\sqrt{17})/2 \approx 2.56$, and θ_i are the roots of $X^3 - X^2 - 3X + 1$ with $\theta_1 \approx 2.17$
- There are not many graphs with PF eigenvalue smaller than 2

For completeness: A formal statement

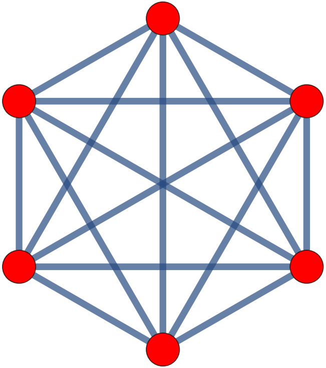
The spectrum contains a lot of properties of the graph (we have already seen some and will see more!)

The spectrum sometimes even determines the graph

- Cospectral = graphs have the same spectrum
- Not quite perfect but expected Cospectral graphs exist; here is an example:

Fig. 1.2 Two cospectral regular graphs
(Spectrum: 4, 1, $(-1)^4$, $\pm\sqrt{5}$, $\frac{1}{2}(1 \pm \sqrt{17})$)

A surprising application of the spectrum



$\mathrm{Aut}(K_n) = S_n$

$\mathrm{Aut}(P_n) = \mathbb{Z}/2\mathbb{Z}$

- If the spectrum is simple, then the graph automorphisms form a 2-group
- Upshot The spectrum contains information about the graph automorphisms

Thank you for your attention!

I hope that was of some help.