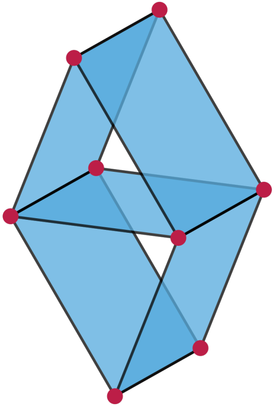
What are...representable matroids 2?

Or: How to rule out matrices

Recognition problems

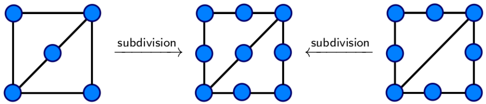
not representable:



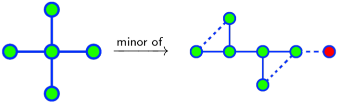
- Linear matroid/representable matroid = obtained from a matrix by taking sets of linearly independent column vectors
- Some matroids are not linear, but how to see this?
- Example The Vámos matroid is not linear – but how can we see this elegantly?

Forbidden strategies

Kuratowski – bottom to top A graph is planar if and only if it does not contain a subgraph which is a subdivision of $K_{3,3}$ or K_5

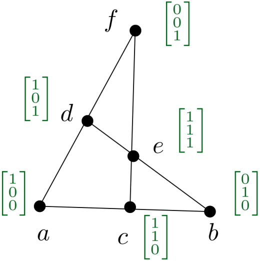


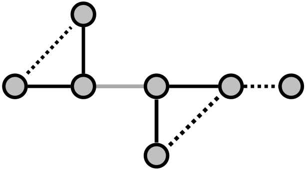
Wagner – top to bottom A graph is planar if and only if it does not contain $K_{3,3}$ or K_5 as a minor



- One can often check a property by checking for nonexistence of certain forbidden things
- Example For planarity the forbidden graphs are the complete graphs K_5 and $K_{3,3}$
- Question Is there something similar for matroids?

Matroid minor





- Think of a matroid as a graph where "lines $\Leftrightarrow$ $\neg$basis"
- Deletions and contractions = delete edges or contract edges
- Minor = obtained by a sequence of deletions and contractions

For completeness: A formal statement

Some representability questions can be attacked by searching for forbidden thingies (F):

- Over arbitrary fields we have F = $U(2,4)$, Fano and dual Fano matroids:

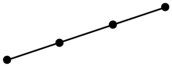
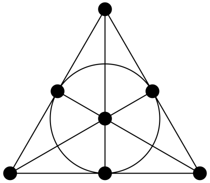


Figure 9: The uniform matroid $U_{2,4}$

Fano:

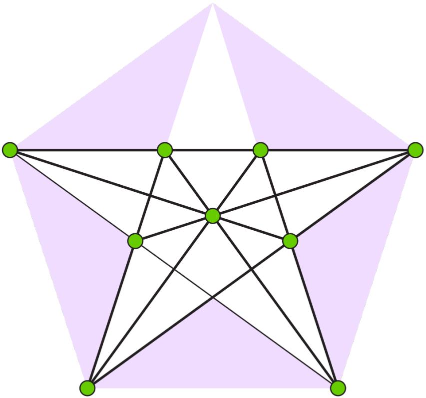


- For $\mathbb{F}_2$ we have F = $U(2,4)$ matroid

Over any finite field there is also a finite list (this is very difficult to prove)

Infinite fields

Perles configuration:



- Perles configuration = nine points and nine lines in $\mathbb{R}^2$ for which every realization has at least one irrational number as a coordinate
- The associated matroid is not representable over $\mathbb{Q}$ but is over $\mathbb{R}$
- In general, for infinite fields no nice forbidden characterization is possible

Thank you for your attention!

I hope that was of some help.