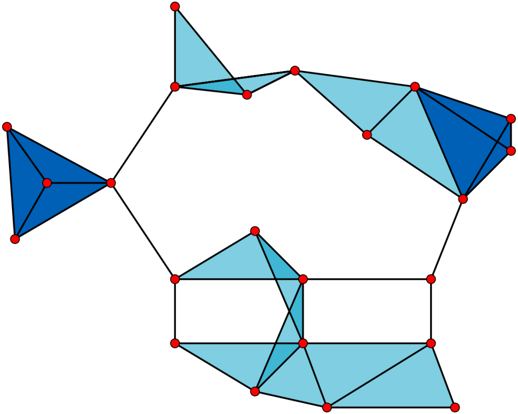
What are...cliques in random graphs?

Or: Peaks!

Complete subgraphs

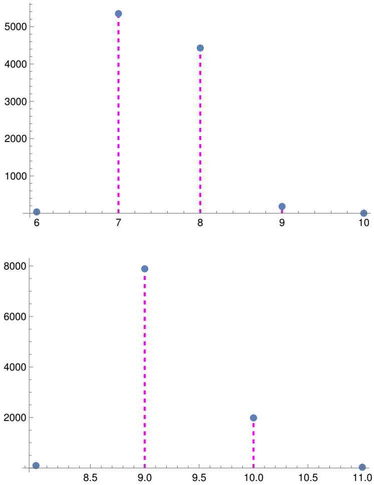


A graph with

- 23 × 1-vertex cliques (the vertices),
- 42 × 2-vertex cliques (the edges),
- 19 × 3-vertex cliques (light and dark blue triangles), and
- 2 × 4-vertex cliques (dark blue areas).

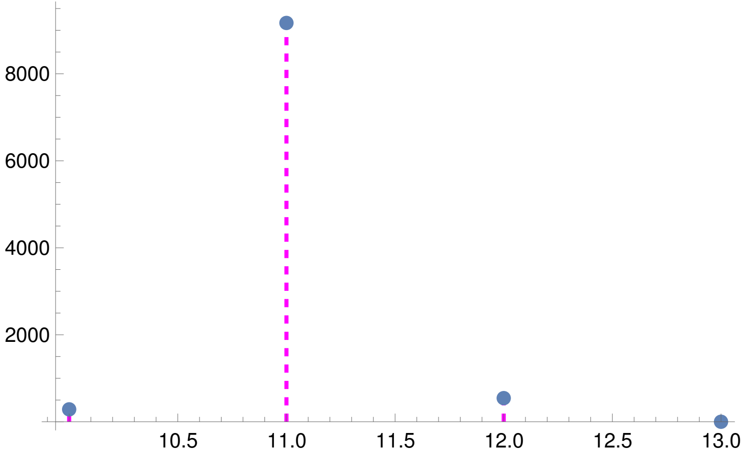
- ▶ Clique = subset of adjacent vertices = complete subgraph
- ▶ Maximal clique = clique that cannot be increased
- ▶ Clique number $cl(G)$ = size of a maximal clique

Two values?



- Above Clique number of 10000 $G_{50,1/2}$ and $G_{100,1/2}$
- There seems to be a concentration around one or two values

Or rather one value!



- Above Clique number of 10000 $G_{200,1/2}$
- There seems to be a peak at one value

For completeness: A formal statement

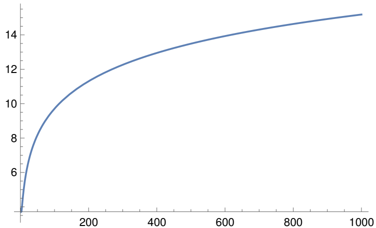
For all $\varepsilon > 0$ we have the probability

$$\lim_{n\to\infty} P(\lfloor f - \varepsilon \rfloor \leq cl(G_{n,p}) \leq \lfloor f + \varepsilon \rfloor) = 1$$

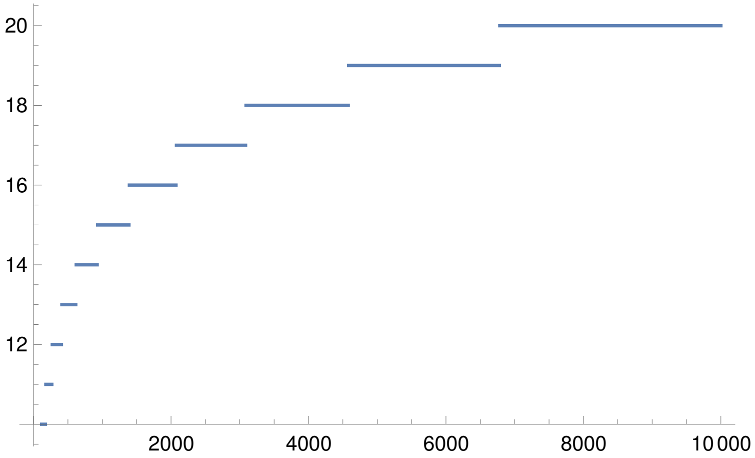
where we have the following threshold function

$$f = 2\log_{1/p}(n) - 2\log_{1/p}\log_{1/p}(n) + \log_{1/p}(e) + 1$$

- This is saying that $cl(G_{n,p}) \approx 2\log_{1/p}(n)$
- Also: $cl(G_{n,p})$ peaks at one or two values, depending on n, p
- Here is a plot for $p = 1/2$ of the threshold function



Staircases



- For $p = 1/2$ we have one peak if $n \gg 0$
- $cl(G_{n,p})$ has a staircase pattern with longer and longer staircases

Thank you for your attention!

I hope that was of some help.