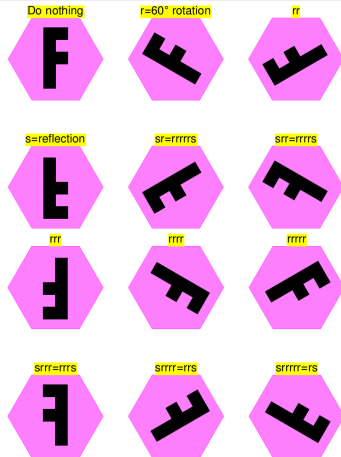


**What are...symmetries of random graphs?**

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Or: There are no symmetries!

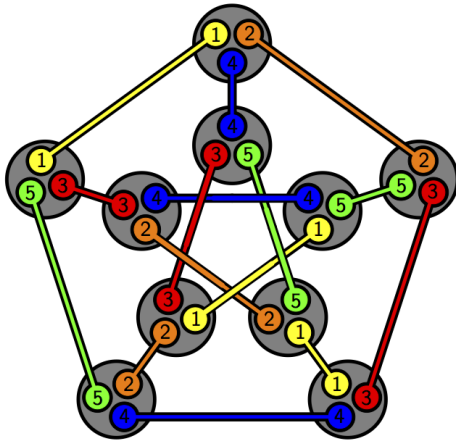
# Symmetries are everywhere



- ▶ Symmetry is a fundamental concept of nature
- ▶ In mathematics symmetry is often measured by symmetry groups
- ▶ Goal Let us explore symmetries of graphs

## Symmetries are everywhere!

$$\text{Aut}(G) = S_5:$$



- ▶  $\text{Aut}(G)$  = set of vertex bijections  $G \rightarrow G$  that preserve connectivity
- ▶ Large  $\text{Aut}(G)$   $\iff$  very symmetric graph
- ▶ Question How symmetric are most graphs?

# Symmetries are everywhere?

*smallest cyclic group graph*  
( $n = 9, |Aut| = 3$ )



*(9,3)-configuration graph 1*  
( $n = 9, |Aut| = 3$ )



*25-Paulus graph 3*  
( $n = 25, |Aut| = 3$ )



*25-Paulus graph 9*  
( $n = 25, |Aut| = 3$ )



*26-Paulus graph 6*  
( $n = 26, |Aut| = 3$ )



*40-fullerene graph 12*  
( $n = 40, |Aut| = 3$ )



*Tutte's graph*  
( $n = 46, |Aut| = 3$ )



*Cremona-Richmond configuration graph*  
( $n = 15, |Aut| = 5$ )



*22-quintic graph 1*  
( $n = 22, |Aut| = 5$ )



*Johnson solid skeleton 47*  
( $n = 35, |Aut| = 5$ )



*Watkins's snark*  
( $n = 50, |Aut| = 5$ )



*42-cubic graph 2*  
( $n = 42, |Aut| = 7$ )



► Naive approach: list all graphs with  $\leq n$  vertices and their  $Aut(G)$

► Observation 1 Most graphs have no symmetries  $Aut(G) \cong 1$

► Observation 2 Some groups appear way more often than other as  $Aut(G)$

## For completeness: A formal statement

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Suppose  $0 < p \leq 1$  and  $M$  are constant, then:

- ▶ Almost all  $G_{n,p}$  have  $Aut(G) \cong 1$
- ▶ Almost all  $G(n, M)$  have  $Aut(G) \cong 1$

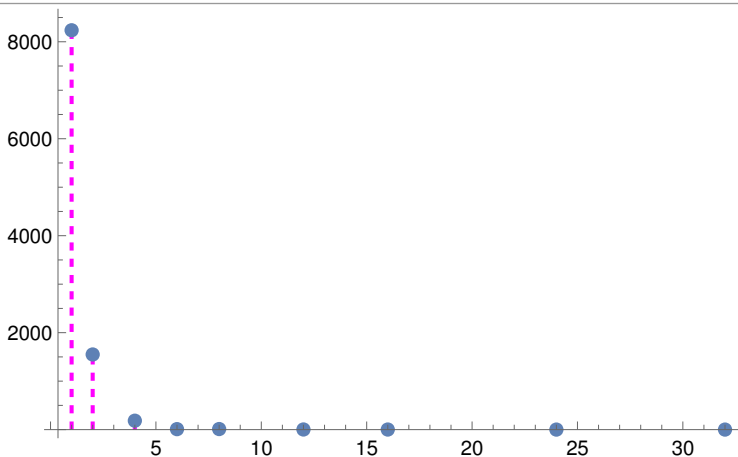
Hence, almost no graph is symmetric

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- ▶ Being asymmetric is the essence of random, so this is actually more way more general (but we can formally prove it for graphs)
- ▶ In contrast, asymmetry in nature is rare and “weird”:



## Some symmetries are super rare



- ▶ Above Mathematica created 10000 random graphs and computed  $|Aut(G)|$
- ▶ Result  $1 \rightarrow 8237, 2 \rightarrow 1550, 4 \rightarrow 184, 6 \rightarrow 10, 8 \rightarrow 12, 12 \rightarrow 3, 16 \rightarrow 2, 24 \rightarrow 1, 32 \rightarrow 1$
- ▶ Some graphs appear much more often than other (can be proven formally)

**Thank you for your attention!**

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I hope that was of some help.