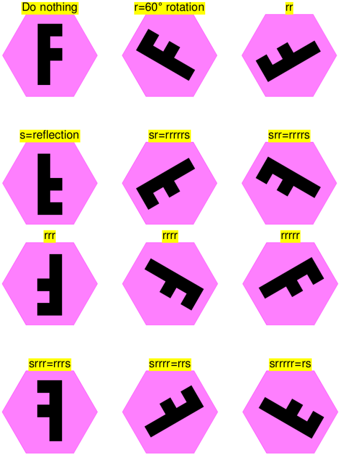
What are...symmetries of random graphs?

Or: There are no symmetries!

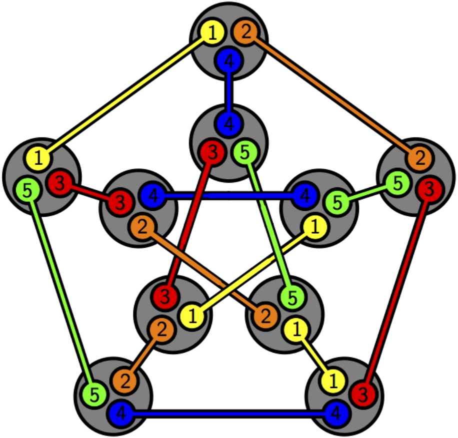
Symmetries are everywhere



- Symmetry is a fundamental concept of nature
- In mathematics symmetry is often measured by symmetry groups
- Goal Let us explore symmetries of graphs

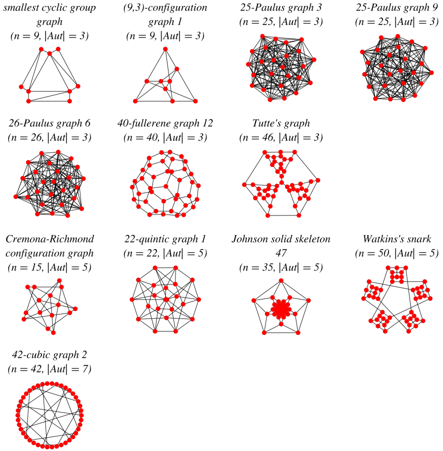
Symmetries are everywhere!

$Aut(G) = S_5$:



- $Aut(G)$ = set of vertex bijections $G \to G$ that preserve connectivity
- Large $Aut(G)$ $\leftrightsquigarrow$ very symmetric graph
- Question How symmetric are most graphs?

Symmetries are everywhere?



- Naive approach: list all graphs with $\leq n$ vertices and their $Aut(G)$
- Observation 1 Most graphs have no symmetries $Aut(G) \cong 1$
- Observation 2 Some groups appear way more often than other as $Aut(G)$

For completeness: A formal statement

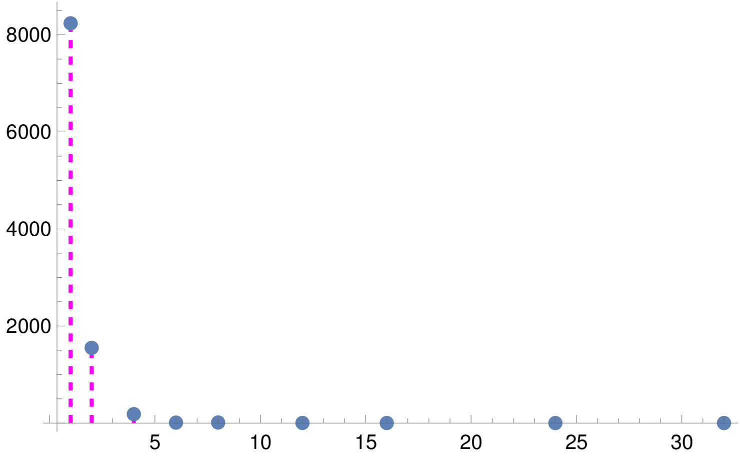
Suppose $0 < p \leq 1$ and M are constant, then:

- Almost all $G_{n,p}$ have $Aut(G) \cong 1$
- Almost all $G(n, M)$ have $Aut(G) \cong 1$

Hence, almost no graph is symmetric

- Being asymmetric is the essence of random, so this is actually more way more general (but we can formally prove it for graphs)
- In contrast, asymmetry in nature is rare and "weird":

Some symmetries are super rare



- Above Mathematica created 10000 random graphs and computed $|Aut(G)|$
- Result $1 \to 8237, 2 \to 1550, 4 \to 184, 6 \to 10, 8 \to 12, 12 \to 3, 16 \to 2, 24 \to 1, 32 \to 1$
- Some graphs appear much more often than other (can be proven formally)

Thank you for your attention!

I hope that was of some help.