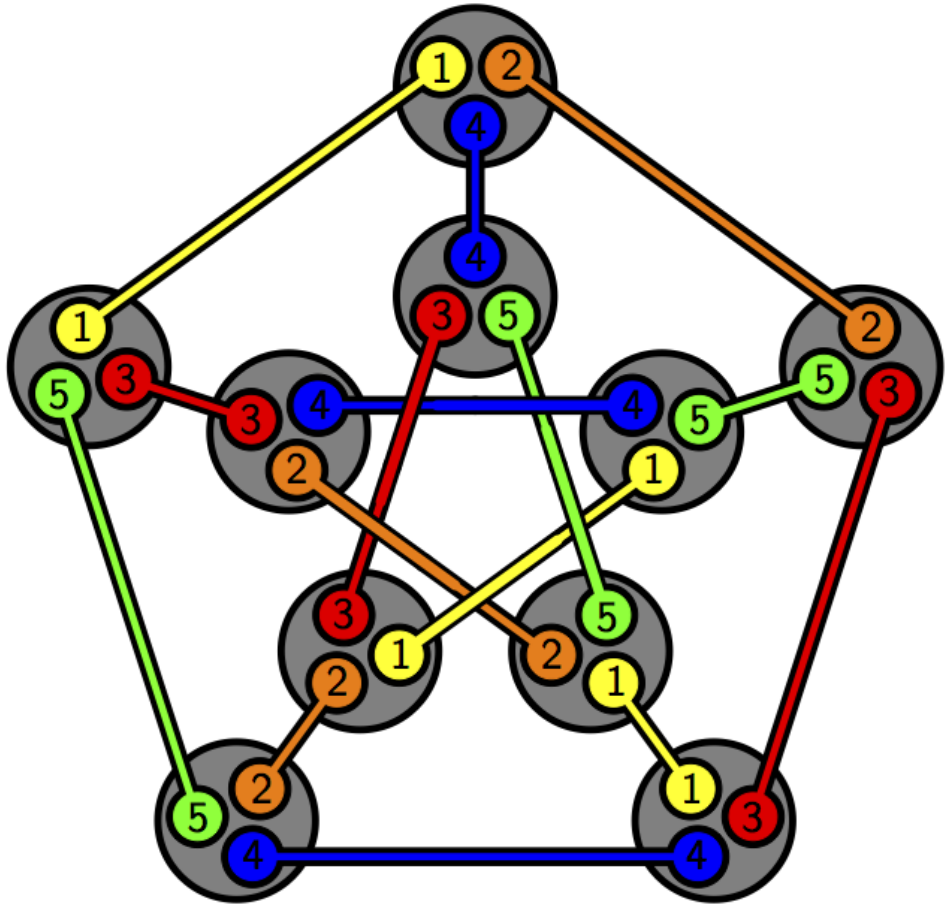
What are...subgraphs of random graphs?

Or: A lot of edges

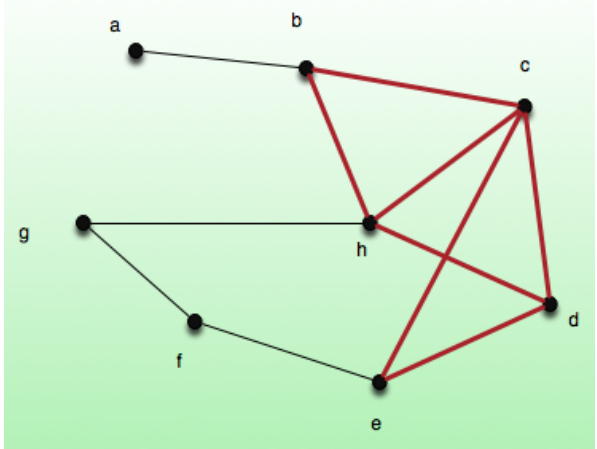
Is your favorite graph a random graph?



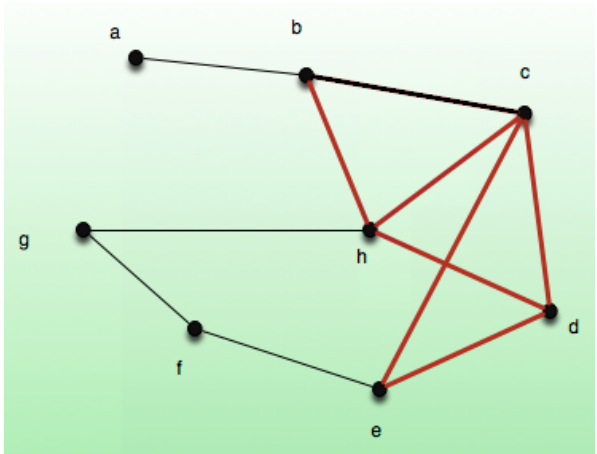
- Fix some graph G "Your favorite"
- Question How likely is G a random graph?
- This question is great but also not great – let us reformulate it!

A maybe better question

Subgraph:

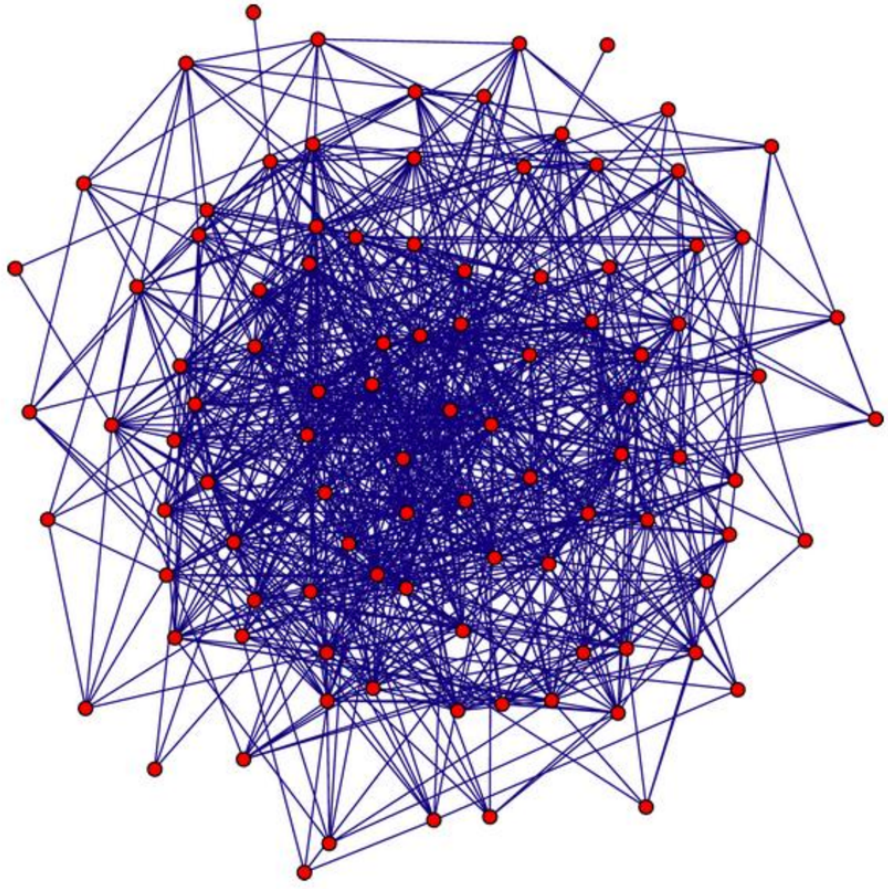


Also a subgraph:



- How often does G appear as a subgraph of a random graph?
- Subgraphs = collection of vertices and fitting edges of a parent graph
- Random graph in this video = $G_{n,p}$ (bottom to top) or $G(n, M)$ (top to bottom)

What should we expect?



- Let us stay with $G_{n,p}$ and $p(n) = p$ constant (more general on the next slide)
- $G_{n,p}$ will contain many edges for $n \gg 0$
- In this case G should appear almost surely

For completeness: A formal statement

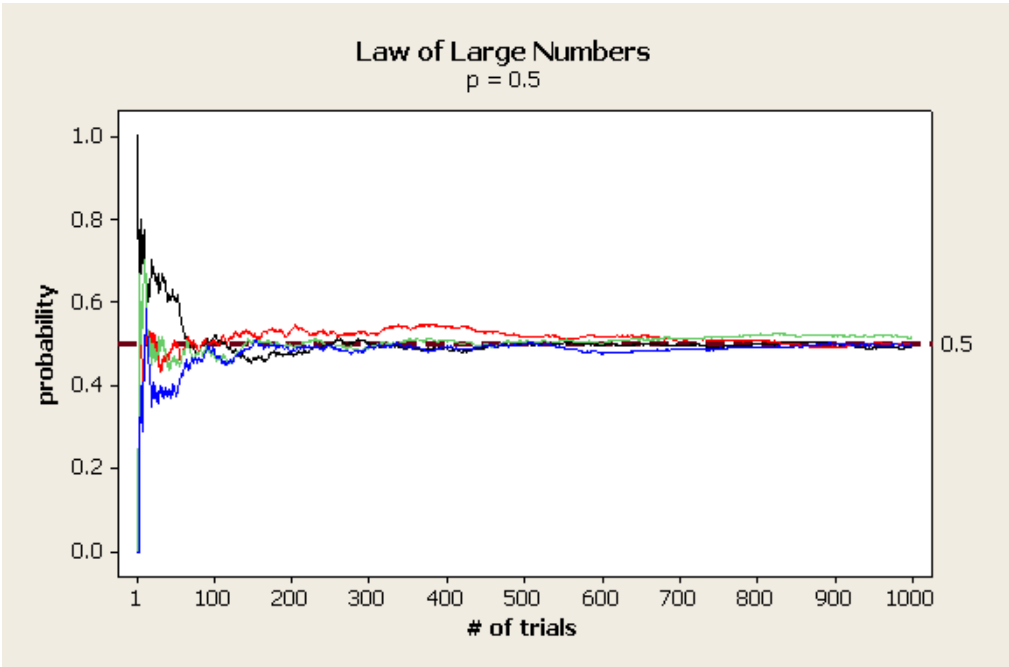
Suppose that for every $\epsilon > 0$ we have

(1) $\lim_{n\to\infty} p(n)n^{\epsilon} = \infty$ and $\lim_{n\to\infty} \big(1 - p(n)\big)n^{\epsilon} = \infty$ Bottom to top

(2) $\lim_{n\to\infty} M(n)n^{\epsilon-2} = \infty$ and $\lim_{n\to\infty} \big(\binom{n}{2} - M(n)\big)n^{\epsilon-2} = \infty$ Top to bottom

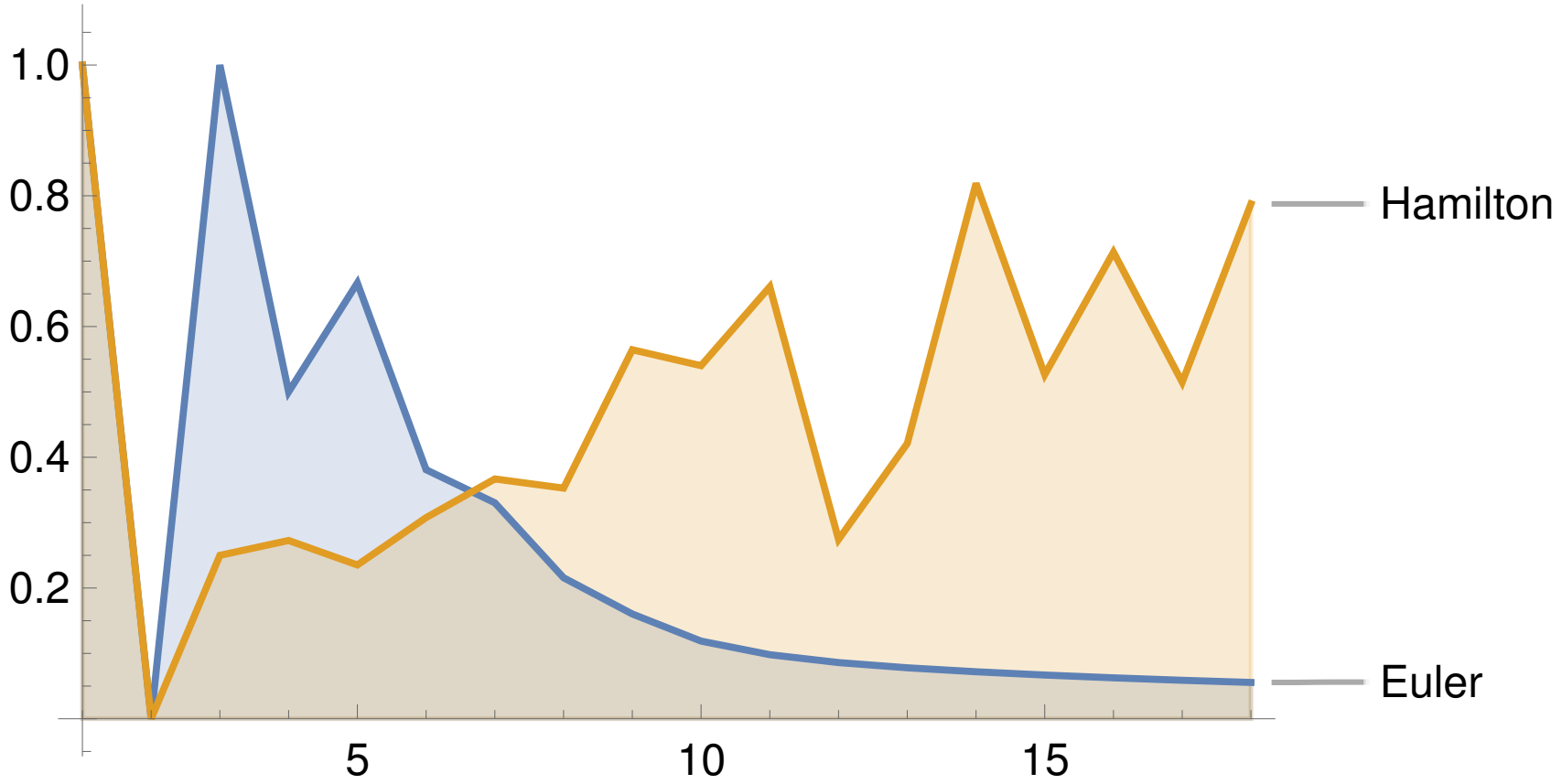
The every G appears almost always in $G_{n,p}$ or $G(n, M)$

- Recall: almost always $\neq$ always!



- The conditions (1) and (2) are quite often satisfied

Slogan: many edges!



- "Real takeaway" Most graphs have many edges
- (Wannabe) consequence Almost all graphs have all properties that are easy to satisfy with many edges - we will explore that!
- Example Almost all graphs are Hamiltonian; above "Hamil/all"

Thank you for your attention!

I hope that was of some help.