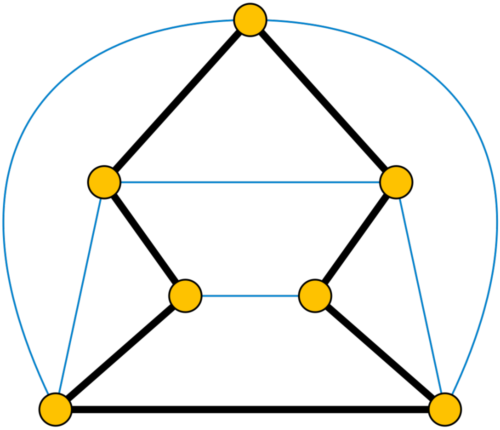
What is...spectral Hamiltonicity?

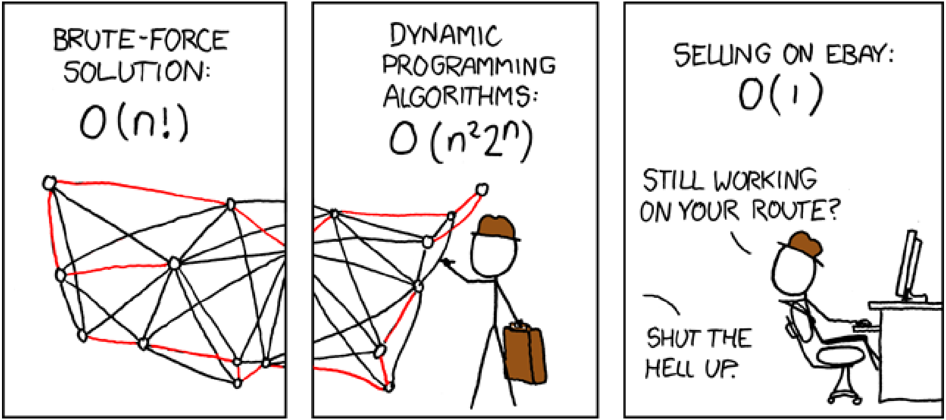
Or: The second largest - part 3

Hamiltonian graphs



- Hamiltonian cycle = a cycle that visits every vertex exactly once
- Hamiltonian graph = a graph with an Hamiltonian cycle
- Question How can we check whether a graph Hamiltonian?

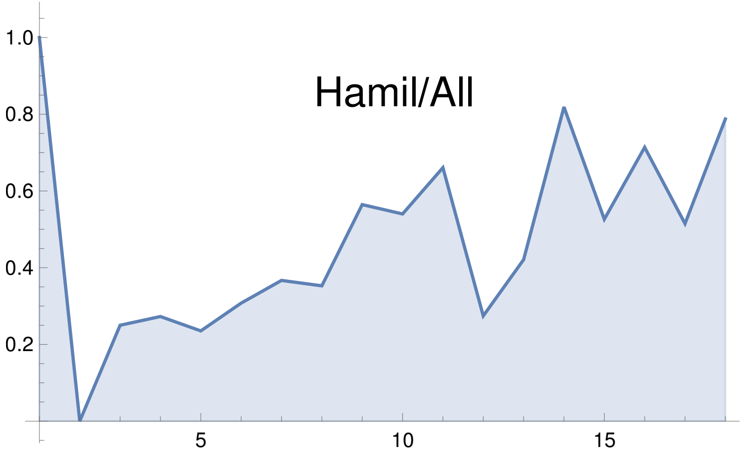
Very difficult



(This is the traveling salesperson problem.)

- Hamiltonian graph was one of the first problems shown to be NP-complete
- NP-complete "=" can't do much better than brute force
- Dynamic programming algorithms solves this is roughly in $O(n^2 2^n)$, $n = \#V$

But almost all graphs are Hamiltonian!



- To determine precisely whether a graph is Hamiltonian is difficult
- To determine approximately whether a graph is Hamiltonian is easy
- Idea Maybe the spectrum helps to prove Hamiltonian for large enough graphs

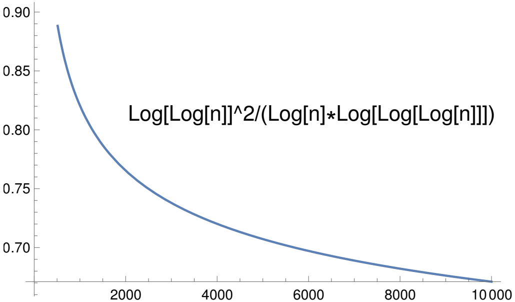
For completeness: A formal statement

For a k-regular graph with n vertices and

$$\lambda_2/\lambda_1 < \frac{(\log\log n)^2}{1000 \log n \log\log\log n}$$

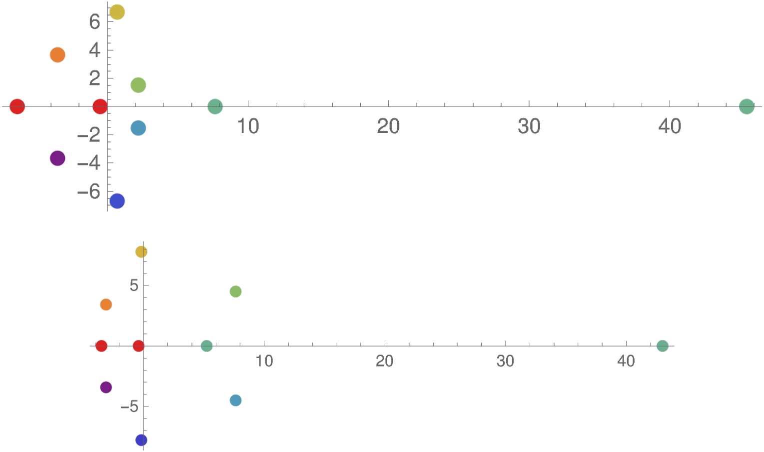
is Hamiltonian

- $\frac{(\log\log n)^2}{1000 \log n \log\log\log n}$ goes to zero but very slowly:



- It follows that every sufficiently large strongly regular graph is Hamiltonian

λ_2 is rather small



- "Very often" $\lambda_2 < 2\sqrt{\lambda_1 - 1} + \varepsilon$
- Thus, λ_2 is "very often" tiny compared to λ_1
- Checking e.g. $\lambda_2/\lambda_1 < 2/n^{1/10}$ for some graphs then implies that they are Hamiltonian

Thank you for your attention!

I hope that was of some help.