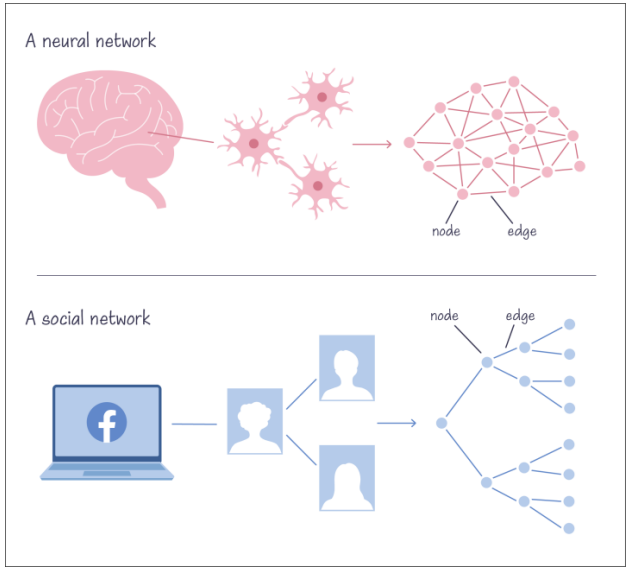
What is...algebraic graph theory?

Or: It's a matrix!

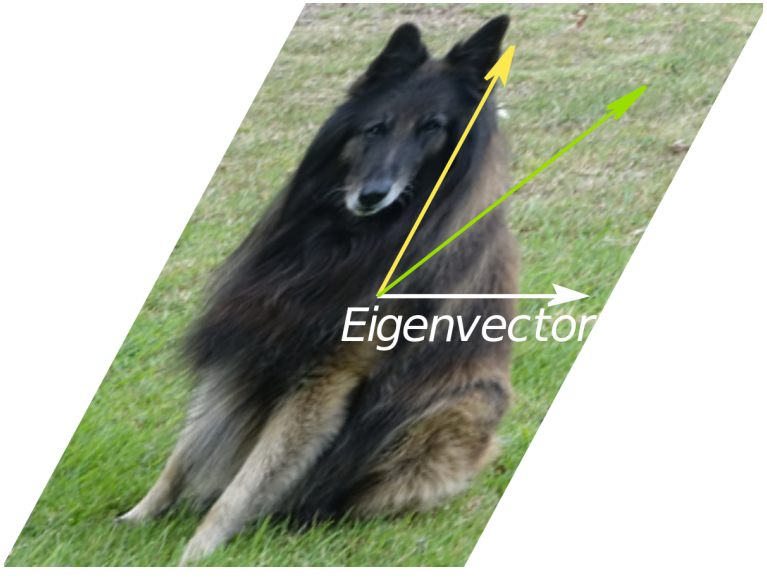
Jumping between fields

From



graph theory

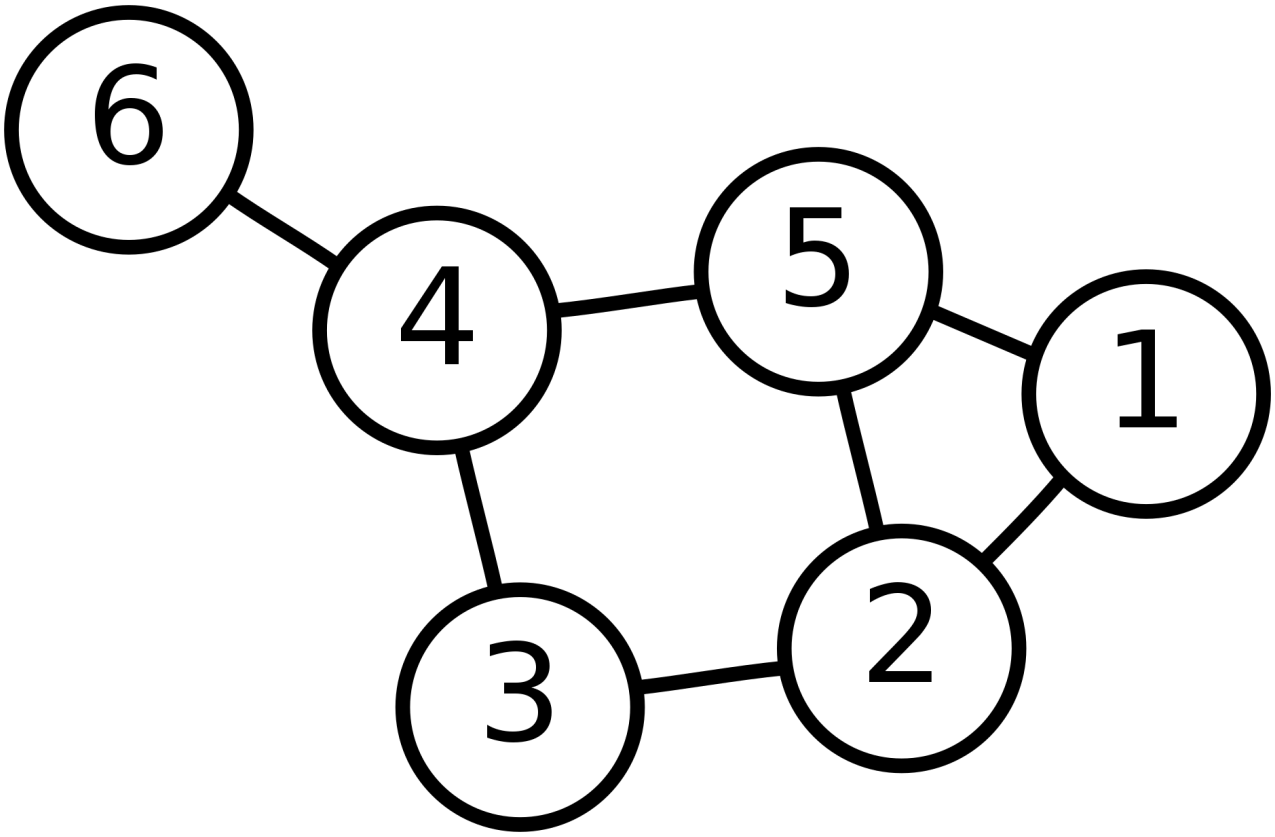
to



algebra and friends

Algebraic graph theory studies discrete objects by using algebraic objects

First main observation



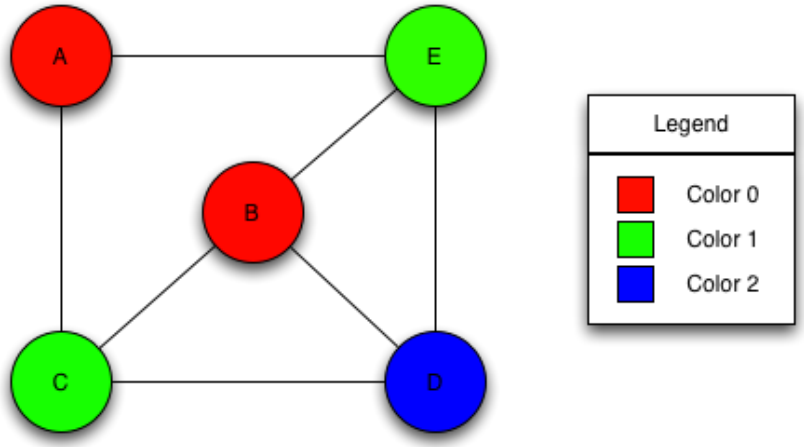
$$\leftrightsquigarrow \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

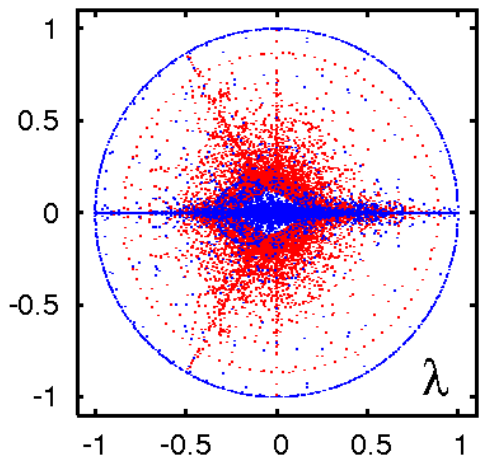
- Underrated fact Graphs "=" matrices (adjacency matrix)
- Essentially every vertex corresponds to a column/row, and edges are entries
- We can thus go back-and-forth between algebraic graph theory and algebra

The keywords – what algebraic graph theory (for example) studies

- Graphs and matrices
  - Adjacency matrix
  - Incidence matrix
  - Laplacian matrix
  - ...
- Apply this to
  - Path, cycles, distance
  - Colorings
  - Random walks
  - ...
- Go deeper into algebra
  - Groups, monoids and graphs
  - Graph polynomials
  - Graph homologies
  - ...

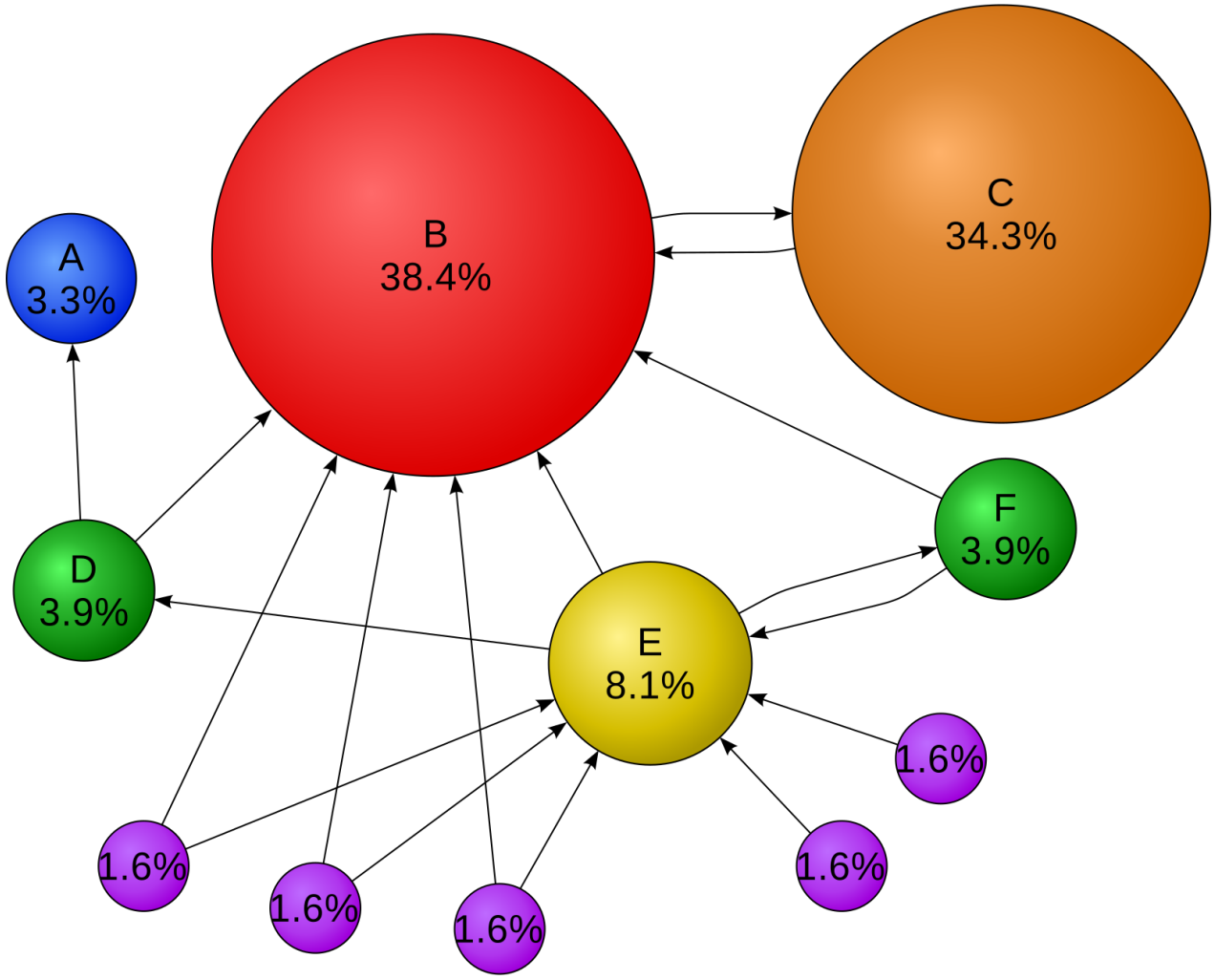
Direction one – graph colorings





- Coloring problems are among the most important problems in mathematics
- They can be attacked using eigenvalues of an adjacency matrix

Direction two – page rank and google



- Page rank lists webpage in a certain order for *e.g.* google
- This works by using the spectrum of an adjacency matrix

Thank you for your attention!

I hope that was of some help.