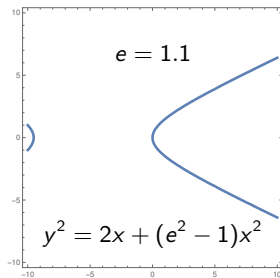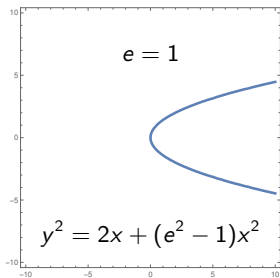**What is...algebraic geometry?**
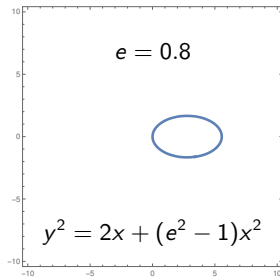
Or: Polynomials, and more

Algebraic geometry (AG) "=" polynomials

Top-left plot:
$e = 0$
$y^2 = 2x + (e^2 - 1)x^2$

Top-right plot:
$e = 0.8$
$y^2 = 2x + (e^2 - 1)x^2$

Bottom-left plot:
$e = 1$
$y^2 = 2x + (e^2 - 1)x^2$

Bottom-right plot:
$e = 1.1$
$y^2 = 2x + (e^2 - 1)x^2$

AG studies solutions to polynomials equations by using algebraic methods
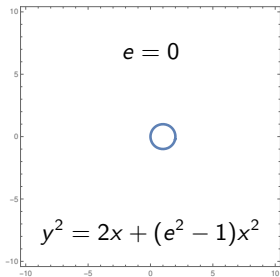
The main fields of AG

The study of polynomials

- circle
- ellipse
- parabola
- hyperbola

Classical alg. geo.

$$x^3 + x^2z^2 - y^2 = 0$$

Modern alg. geo.

GENERIC PTS OVER PRIMES P

CONJUGACY CLASSES OF ALG. NOS OVER Z[1/n]

(2)   (3)   (7)   (P)   (1)
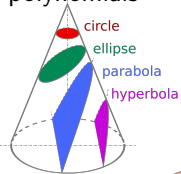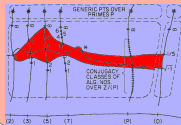
Modern v2 alg. geo.
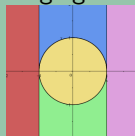
Classical⤳varieties+friends , modern⤳schemes+friends , modern v2⤳Gröbner+friends

The keywords – what AG (for example) studies

- Classical
  - ▷ Affine varieties

  - ▷ Projective varieties

  - ▷ Smooth varieties

  - ▷ ...

- Modern
  - ▷ Schemes

  - ▷ Sheaves

  - ▷ Stacks

  - ▷ ...

- Modern v2
  - ▷ Gröbner bases

  - ▷ Homotopy continuation

  - ▷ Varieties and friends over finite fields

  - ▷ ...

Direction one − AG in robotics + friends
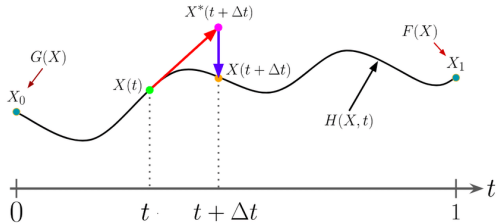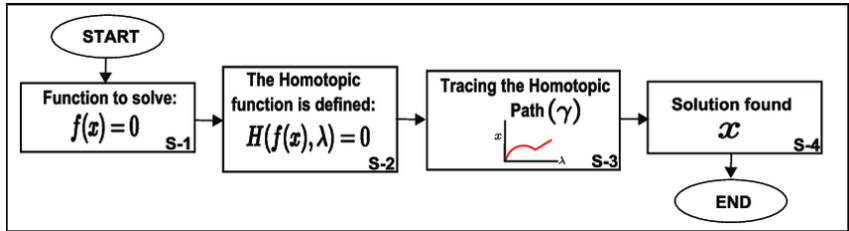
- Homotopy continuation = solving polynomial equations by tracking the solutions of "nearby" and "easier" polynomial equations

- This has found applications in robotics, chemistry etc.
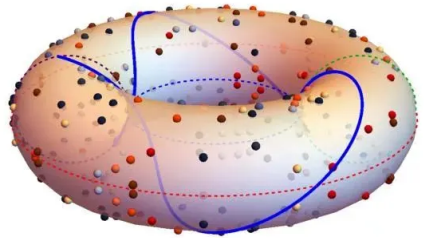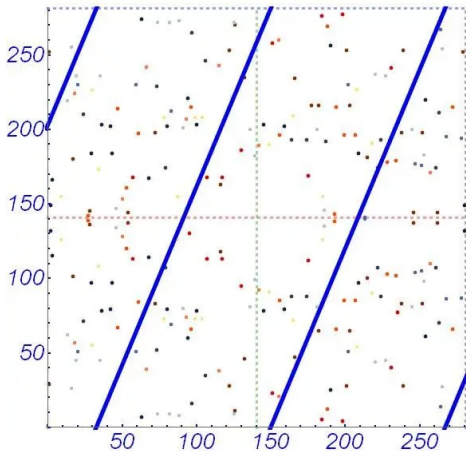
- Elliptic curve $=$ variety that behaves like a torus and has an addition

- Elliptic curves over finite fields play a key role in cryptography

Thank you for your attention!

I hope that was of some help.