## EXERCISES 8: LECTURE REPRESENTATION THEORY

**Exercise 1.** Show that the representation induced from the trivial representation of the trivial subgroup of $G$ is the regular representation of $G$.

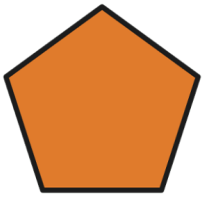
**Exercise 2.** Here are also the character tables of $\mathbb{Z}/5\mathbb{Z}$ and $D_5$:

```
------------------------------
Class |   1   2   3   4   5
Size  |   1   1   1   1   1
Order |   1   5   5   5   5
------------------------------
p  =  5   1   1   1   1   1
------------------------------
X.1   +   1   1   1   1   1
X.2   0   1  Z1 Z1#2 Z1#3 Z1#4
X.3   0   1 Z1#4 Z1#3 Z1#2  Z1
X.4   0   1 Z1#3  Z1 Z1#4 Z1#2
X.5   0   1 Z1#2 Z1#4  Z1 Z1#3
```

```
----------------------
Class |   1  2   3   4
Size  |   1  5   2   2
Order |   1  2   5   5
----------------------
p  =  2   1  1   4   3
p  =  5   1  2   1   1
----------------------
X.1   +   1  1   1   1
X.2   +   1 -1   1   1
X.3   +   2  0  Z1 Z1#2
X.4   +   2  0 Z1#2  Z1
```

Identify the representations induced from the simple $\mathbb{Z}/5\mathbb{Z}$ representations to $D_5$.

**Exercise 3.** $\mathbb{Z}/5\mathbb{Z}$ and $D_5$ act on the pentagon:

$\mathbb{Z}/5\mathbb{Z}$ acts by rotation on and $D_5$ acts by rotation/reflection on 

Identify the representation induced from the rotation action of $\mathbb{Z}/5\mathbb{Z}$ to $D_5$.

**Exercise 4.** Here are the character tables of $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ and the quaternion group $Q_8$:

```
--------------------
Class |   1  2  3  4
Size  |   1  1  1  1
Order |   1  2  2  2
--------------------
p  =  2   1  1  1  1
--------------------
X.1   +   1  1  1  1
X.2   +   1 -1  1 -1
X.3   +   1  1 -1 -1
X.4   +   1 -1 -1  1
```

## Character table of $Q_8$

$Q_8$: Quaternion group; = $C_4.C_2$ = $Dic_2$ = $2_-^{1+2}$

| class | 1 | 2 | 4A | 4B | 4C | |
|---|---|---|---|---|---|---|
| size | 1 | 1 | 2 | 2 | 2 | |
| $\rho_1$ | 1 | 1 | 1 | 1 | 1 | trivial |
| $\rho_2$ | 1 | 1 | -1 | 1 | -1 | linear of order 2 |
| $\rho_3$ | 1 | 1 | 1 | -1 | -1 | linear of order 2 |
| $\rho_4$ | 1 | 1 | -1 | -1 | 1 | linear of order 2 |
| $\rho_5$ | 2 | -2 | 0 | 0 | 0 | symplectic faithful, Schur index 2 |

Identify the representations induced from the simple $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ representations to $Q_8$.

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- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-rt-2022.html.
- Slogan: "Everything that could be finite is finite, unless stated otherwise.". For example, groups are finite and representations are on finite dimensional vector spaces.
- There might be typos on the exercise sheets, my bad, so be prepared.