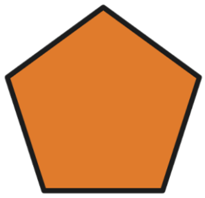
EXERCISES 4: LECTURE REPRESENTATION THEORY

Exercise 1. Here is the character table of $\mathbb{Z}/5\mathbb{Z}$:

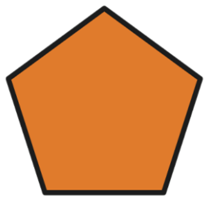
		1	2	3	4	5
Class	|	1	2	3	4	5
Size	|	1	1	1	1	1
Order	|	1	5	5	5	5
p =	5	1	1	1	1	1
X.1	+	1	1	1	1	1
X.2	0	1	Z1	Z1#2	Z1#3	Z1#4
X.3	0	1	Z1#4	Z1#3	Z1#2	Z1
X.4	0	1	Z1#3	Z1	Z1#4	Z1#2
X.5	0	1	Z1#2	Z1#4	Z1	Z1#3

In this table $Z1$ is a primitive 5th root of unity. Use this table to decompose the representation of $\mathbb{Z}/5\mathbb{Z}$ on the pentagon

$\mathbb{Z}/5\mathbb{Z}$ acts by rotation on 

given by rotation of five vertices (so on $\mathbb{C}^5$).

Exercise 2. In Exercise 1, what changes if one uses the rotation/reflection action of D_5 (this group has ten elements)?

D_5 acts by rotation/reflection on 

Here is the relevant character table:

Class	|	1	2	3	4
Size	|	1	5	2	2
Order	|	1	2	5	5
p =	2	1	1	4	3
p =	5	1	2	1	1
X.1	+	1	1	1	1
X.2	+	1	-1	1	1
X.3	+	2	0	Z1	Z1#2
X.4	+	2	0	Z1#2	Z1

Exercise 3. Here is the character table of S_3:

```
-----------------
Class |   1  2  3
Size  |   1  3  2
Order |   1  2  3
-----------------
p  =  2   1  1  3
p  =  3   1  2  1
-----------------
X.1    +   1  1  1
X.2    +   1 -1  1
X.3    +   2  0 -1
```

How does the tensor product of the two-dimensional simple S_3-module with itself decompose?

Exercise 4. Let $\mathrm{SL}_2(\mathbb{F}_q)$ be the finite group of 2x2 matrices of determinant one and entries in the finite field $\mathbb{F}_q$. Convince yourself that $\mathrm{SL}_2(\mathbb{F}_2) \cong S_3$ as groups, and compute the number of elements of $\mathrm{SL}_2(\mathbb{F}_4)$ using its character table:

```
--------------------------
Class |   1  2  3    4    5
Size  |   1 15 20   12   12
Order |   1  2  3    5    5
--------------------------
p  =  2   1  1  3    5    4
p  =  3   1  2  1    5    4
p  =  5   1  2  3    1    1
--------------------------
X.1    +   1  1  1    1    1
X.2    +   3 -1  0   Z1 Z1#2
X.3    +   3 -1  0 Z1#2   Z1
X.4    +   4  0  1   -1   -1
X.5    +   5  1 -1    0    0
```

Decompose the tensor product of the representations associated to χ_4 and χ_5.

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-rt-2022.html.
- Slogan: "Everything that could be finite is finite, unless stated otherwise.". For example, groups are finite and representations are on finite dimensional vector spaces.
- There might be typos on the exercise sheets, my bad, so be prepared.