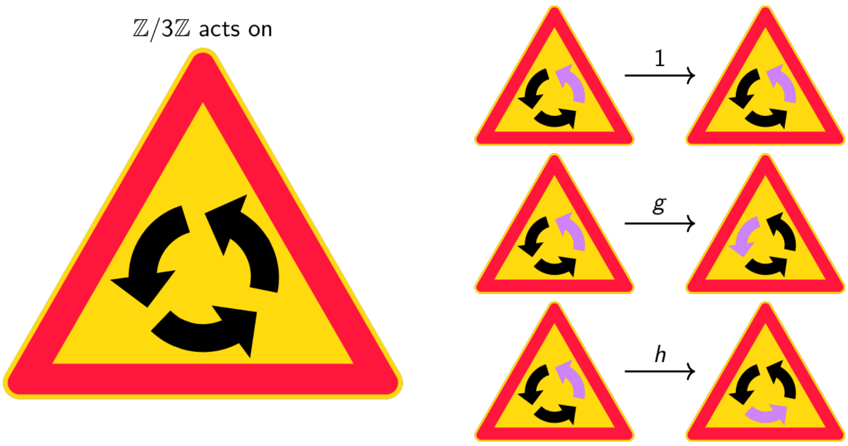
EXERCISES 2: LECTURE REPRESENTATION THEORY

Exercise 1. Show that a representation ϕ of dimension two is simple if and only if there is no common eigenvector to all ϕ_g with $g \in G$.

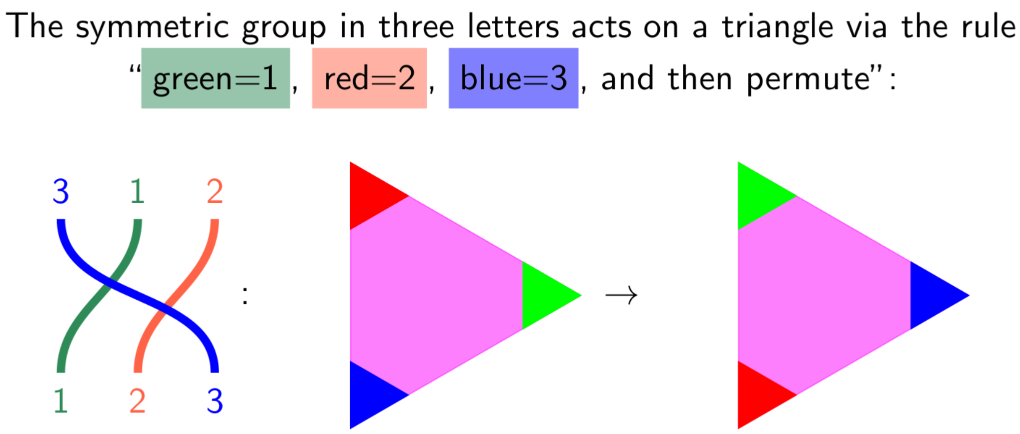
Exercise 2. Recall the module V of $\mathbb{Z}/n\mathbb{Z}$ via the rotation action on an $n$gon.



$$\mathbb{Z}/3\mathbb{Z} = \{1, g, h | g^2 = h, g^3 = 1\}.$$

Show that V is not simple.

Exercise 3. S_3 acts on a triangle by permuting the vertices:



Show that the associated S_3-module, call it V_{perm}, has a one-dimensional subrepresentation V_{triv}.

Exercise 4. Show that $V_{\text{perm}}/V_{\text{triv}}$ from Exercise 4 is simple.

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-rt-2022.html.
- Slogan: "Everything that could be finite is finite, unless stated otherwise.". For example, groups are finite and representations are on finite dimensional vector spaces.
- There might be typos on the exercise sheets, my bad, so be prepared.