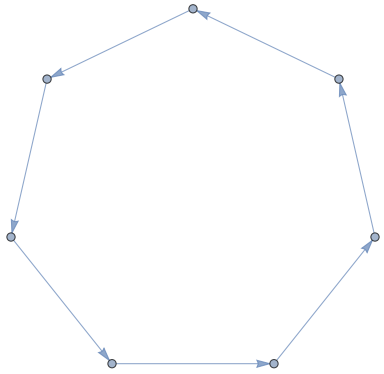
EXERCISES 12: LECTURE REPRESENTATION THEORY

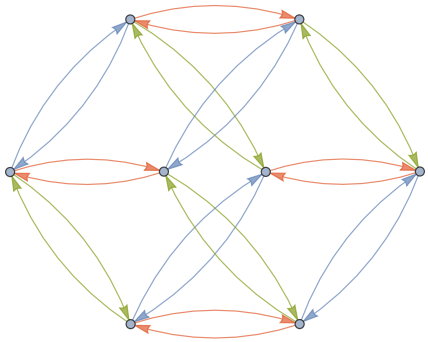
Exercise 1. Here is the Cayley graph of $\mathbb{Z}/7\mathbb{Z}$:



Show that the simple random walk (picking all adjacent edges equally likely for the next step) on this graph is ergodic. What happens for $\mathbb{Z}/8\mathbb{Z}$ instead of $\mathbb{Z}/7\mathbb{Z}$?

Recall: A random walk on a group G driven by a probability P is said to be ergodic if there exists an integer $N > 0$ such that $P^{*N}(g) > 0$ for all $g \in G$.

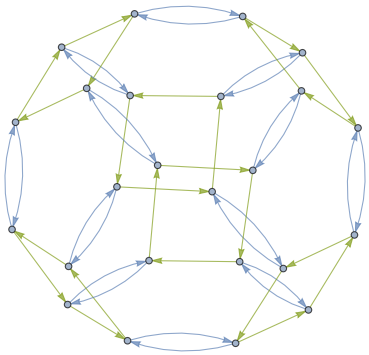
Exercise 2. Here is the Cayley graph of $(\mathbb{Z}/2\mathbb{Z})^3$:



Decide whether the simple random walk on this graph is ergodic.

For simple random walk and ergodic see Exercise 1.

Exercise 3. Here is the Cayley graph of S_4:



Decide whether the simple random walk on this graph is ergodic.

For simple random walk and ergodic see Exercise 1.

Exercise 4. Repeat Exercises 1, 2 and 3 for the lazy random walk on these Cayley graphs.

Recall: A random walk is lazy if 50% of the time one doesn't move and 50% of the time an adjacent edge is picked with equal probability.

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-rt-2022.html.
- Slogan: "Everything that could be finite is finite, unless stated otherwise.". For example, groups are finite and representations are on finite dimensional vector spaces.
- There might be typos on the exercise sheets, my bad, so be prepared.