

EXERCISES 11: LECTURE REPRESENTATION THEORY

Exercise 1. Classify the simple $C_{i,p}$ -representations over \mathbb{C} . What about other fields? (For $C_{i,p}$ see the previous exercise sheet.)

Hint: The cell structure of $C_{3,2}$ is

$$\begin{array}{lcl}
 \mathcal{J}_t & a^3, a^4 & \mathcal{H}(e) \cong \mathbb{Z}/2\mathbb{Z} \\
 \mathcal{J}_{a^2} & a^2 & \\
 \mathcal{J}_a & a & \\
 \mathcal{J}_b & 1 & \mathcal{H}(e) \cong 1
 \end{array}$$

Exercise 2. Classify the simple T_3 -representations over \mathbb{C} . What about other fields? (For T_3 see the previous exercise sheet.)

Hint: The cell structure of T_3 is

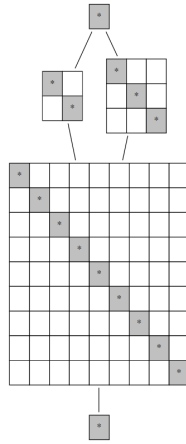
$$\begin{array}{lcl}
 \mathcal{J}_t & \begin{array}{c} (111) \\ (222) \\ (333) \end{array} & \mathcal{H}(e) \cong S_1 \\
 \mathcal{J}_m & \begin{array}{|c|c|c|} \hline (122), (211) & (121), (212) & (221), (112) \\ \hline (133), (311) & (313), (131) & (113), (331) \\ \hline (233), (322) & (323), (232) & (223), (332) \\ \hline \end{array} & \mathcal{H}(e) \cong S_2 \\
 \mathcal{J}_b & \begin{array}{|c|} \hline (123), (213), (132) \\ \hline (231), (312), (321) \\ \hline \end{array} & \mathcal{H}(e) \cong S_3
 \end{array}$$

Exercise 3. Classify the simple TL_n -representations over arbitrary fields. (For TL_n see the previous exercise sheet.)

Hint: The cell structure of TL_4 is

$$\begin{array}{lcl}
 \mathcal{J}_0 & \begin{array}{|c|c|} \hline \text{Diagram 1} & \text{Diagram 2} \\ \hline \end{array} & \mathcal{H}(e) \cong 1 \\
 \mathcal{J}_2 & \begin{array}{|c|c|c|} \hline \text{Diagram 3} & \text{Diagram 4} & \text{Diagram 5} \\ \hline \end{array} & \mathcal{H}(e) \cong 1 \\
 \mathcal{J}_4 & \begin{array}{|c|c|c|} \hline \text{Diagram 6} & \text{Diagram 7} & \text{Diagram 8} \\ \hline \end{array} & \mathcal{H}(e) \cong 1
 \end{array}$$

Exercise 4. A not further specified monoid S has cell structure



with gray boxes indicating idempotent \mathcal{H} cells.

What can you say about the simple S -representations?

- ▶ The exercises are optimal and not mandatory. Still, they are highly recommend.
- ▶ There will be 12 exercise sheets, all of which have four exercises.
- ▶ The sheets can be found on the homepage www.dtubbenhauer.com/lecture-rt-2022.html.
- ▶ Slogan: “Everything that could be finite is finite, unless stated otherwise.”. For example, groups are finite and representations are on finite dimensional vector spaces.
- ▶ There might be typos on the exercise sheets, my bad, so be prepared.