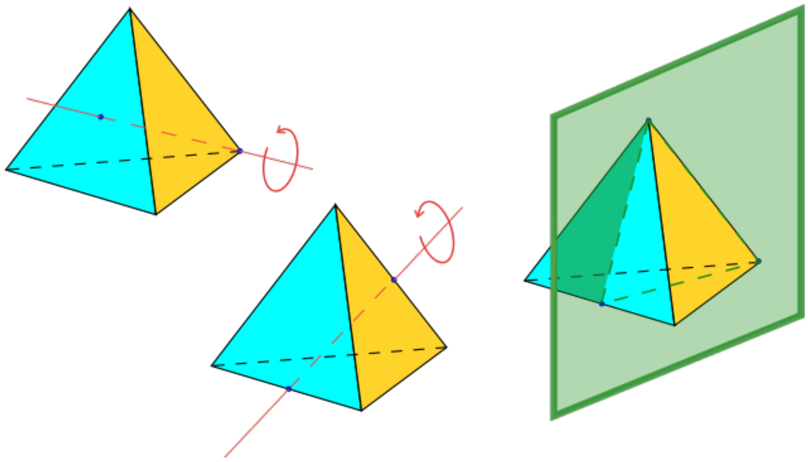
EXERCISES 1: LECTURE REPRESENTATION THEORY

Exercise 1. For any set X, let $G = \mathrm{Aut}(X)$ the group of automorphisms of X. Endow $\mathbb{K}X$ with the structure of a G-module, where $\mathbb{K}$ is an arbitrary field.

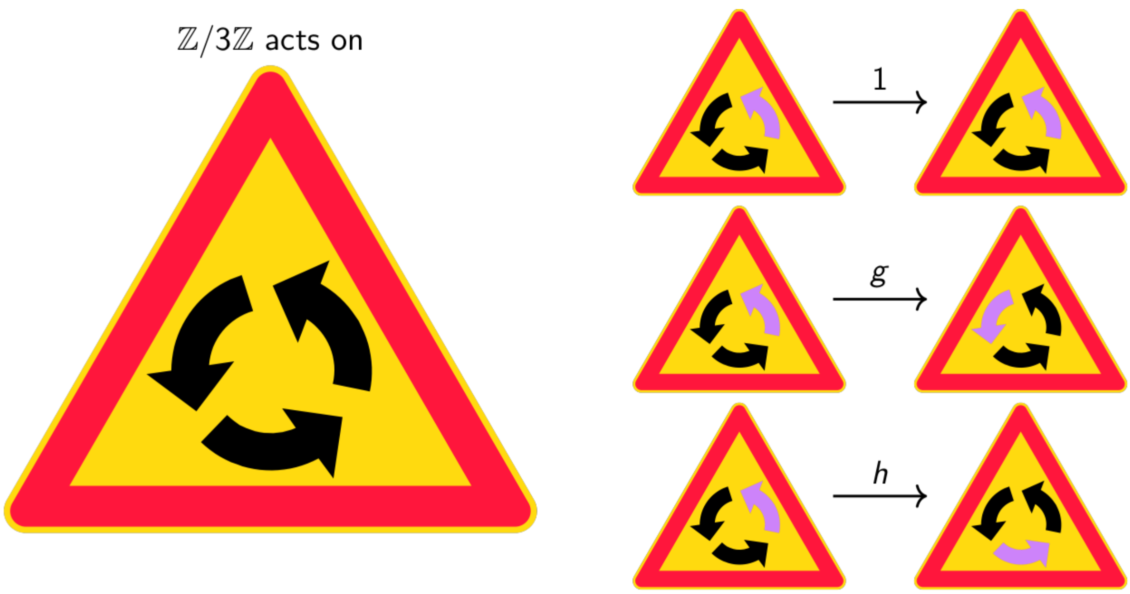
Exercise 2. An example of Exercise 1 is the symmetric group S_n on $\{1, \dots, n\}$ acting by permutation on an $(n-1)$-simplex. For example, when $n = 4$ the group S_4 acts on the tetrahedron:

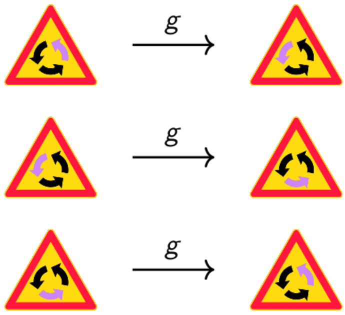


The three displayed actions are the ones from (234) (left), (12)(34) (middle) and (34) when one labels the vertices 1 (right), 2 (top), 3 (front) and 4 (left).

After extending scalars to $\mathbb{K}$, show that this representation always has a S_n-invariant subspace (so a subrepresentation).

Exercise 3. Show that the rotational symmetries of $n$gons can be interpreted as a representation of $\mathbb{Z}/n\mathbb{Z}$ on $\mathbb{C}^3$.





$$g \leftrightsquigarrow \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Exercise 4. For $k \in \mathbb{Z}$ show that $\phi_k \colon \mathbb{Z}/n\mathbb{Z} \to \mathbb{C}$ determined by $\phi(1) = \exp(2\pi i k/n)$ defines a representation of $\mathbb{Z}/n\mathbb{Z}$. When are ϕ_k and ϕ_l equivalent?

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-rt-2022.html.
- Slogan: "Everything that could be finite is finite, unless stated otherwise.". For example, groups are finite and representations are on finite dimensional vector spaces.
- There might be typos on the exercise sheets, my bad, so be prepared.