LECTURE: REPRESENTATION THEORY

Disclaimer

Nobody is perfect, and I might have written or said something silly. If there is any doubt, then please check the references or contact me. All questions welcome!

Who?

Fourth semester students in Mathematics interested in a mixture of (linear) algebra, group theory and discrete mathematics, but everyone is welcome.

Where and when?

- The lecture.
  - Every Monday from 12:00–14:00.
  - Online.
- The tutorials.
  - Every Friday from 12:00–14:00.
  - Online.

Material for the lecture

- The lecture is a mix of various sources for group and monoid representations. The main source is [St12] for group representations and then [St16] for the monoid case, and the lecture follows the list of topics presented therein.

  The lecture sometimes takes a different perspective and potentially reading either of the classical references [CR62], [FH91] or [Se77] should be beneficial. [Be98] is a bit more abstract, but also a classic. Newer references are for example [Cr19], [E+11] (freely available), [Sa01] (for symmetric groups). These are also used for the lecture.
- Website www.dtubbenhauer.com/lecture-rt-2022.html
- Prerecorded lectures on the "What is...representation theory?" playlist here: www.youtube.com/c/VisualMath/playlists
- Exercise sheets are available on the course website.

Schedule.

- The beginnings – What is...representation theory?
- Simple and indecomposable representations I – The elements.
- Simple and indecomposable representations II – More about elements.
- Characters I – The main players of representation theory!?
- Characters II – Schur's orthogonality relations.
- Characters III – Abelian groups and Fourier analysis.
- Burnside's theorem – An application.
- Induction and restriction – The classical adjoint pair.
- Representations of symmetric groups – Young diagrams and co.
- Monoids I – Green's relations and friends.
- Monoids II – The Clifford–Munn–Ponizovskiĭ theorem.

- Whats next? – Outlook.

References

[Be98] D.J. Benson. Representations and cohomology. I. Basic representation theory of finite groups and associative algebras. Second edition. Cambridge Studies in Advanced Mathematics, 30. Cambridge University Press, Cambridge, 1998. xii+246 pp.

[Cr19] D.A. Craven. Representation theory of finite groups: a guidebook. Universitext. Springer, Cham, 2019. viii+294 pp.

[CR62] C.W. Curtis, I. Reiner. Representation theory of finite groups and associative algebras. Reprint of the 1962 original. AMS Chelsea Publishing, Providence, RI, 2006. xiv+689 pp.

[E+11] P. Etingof, O. Golberg, S. Hensel, T. Liu, A. Schwendner, D. Vaintrob, E. Yudovina. Introduction to representation theory. With historical interludes by Slava Gerovitch. Student Mathematical Library, 59. American Mathematical Society, Providence, RI, 2011. viii+228 pp. https://math.mit.edu/~etingof/replect.pdf

[FH91] W. Fulton, J. Harris. Representation theory. A first course. Graduate Texts in Mathematics, 129. Readings in Mathematics. Springer-Verlag, New York, 1991. xvi+551 pp.

[Sa01] B.E. Sagan. The symmetric group. Representations, combinatorial algorithms, and symmetric functions. Second edition. Graduate Texts in Mathematics, 203. Springer-Verlag, New York, 2001. xvi+238 pp.

[Se77] J.P. Serre. Linear representations of finite groups. Translated from the second French edition by Leonard L. Scott. Graduate Texts in Mathematics, Vol. 42. Springer-Verlag, New York-Heidelberg, 1977. x+170 pp.

[St12] B. Steinberg. Representation theory of finite groups. An introductory approach. Universitext. Springer, New York, 2012. xiv+157 pp.

[St16] B. Steinberg. Representation theory of finite monoids. Universitext. Springer, Cham, 2016. xxiv+317 pp.

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