Potential master or Ph.D. project of I WANT YOU in 2023

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Key information

Candidate. I WANT YOU.

Email. YOUR EMAIL.

Research areas. Algebra, representation theory, and topology, more specifically knot polynomials and their representation theoretical incarnations.

Title. *"Symmetric webs and colored Jones polynomials"*.

First read. [Kup96].

A classical result of Rumer, Teller, and Weyl, modernly interpreted, states that the Temperley–Lieb category describes the full subcategory of quantum sl_2-modules generated by tensor products of the two-dimensional vector representation of quantum sl_2. The former was first introduced in the study of statistical mechanics (as an algebra and also in the non-quantum setting) by Temperley and Lieb and has played an important role in several areas of mathematics and physics.

More generally a web category is an axiomatization of the representation theory of a group, quantum group, Lie algebra, or other group or group-like object. Being diagrammatic in nature, having a web category gives a method to solve questions in representation theory by purely graphical and combinatorial ideas.

Further, there is often a braid group action on such web categories, making an immediate bond to low-dimensional topology. For example, the Temperley–Lieb category, a.k.a. the sl_2-web category, gives a way to compute the famous Jones polynomial of a knot. Even better, web categories give such knot polynomials a home in a whole zoo of knot, 3- and 4-manifold invariants obtained via representation theory, "explaining their existence".

One cousin of the Temperley–Lieb category, a.k.a. symmetric webs [RT16], gives an easy and hands-on way to understand the whole representation category of sl_2, as well as the colored Jones polynomials.

Minimal goal. Summarize the paper of about symmetric webs in your own words.
Average goal. Add a more detailed discussion about colored Jones polynomials and their computations.
Optimal goal (for Ph.D.). Address some of the open questions mentioned below.
Key. Be concise with the basics. Be precise with the categorical actions. Find a self-containing way to summarize various results scattered over the literature.

The thesis in details – minimal goal

The minimal master project should be structured as follows.

- Write an introduction and explain the main ideas of quantum topology, see *e.g.* [Tur94, Introduction]. Explain how your mater thesis fits into this framework.
- Summarize basics about diagrammatic algebra, see *e.g.* [TV17, Chapter I].
- Explain the symmetric web calculus in details.
- Explain the main theorem in [RT16].

The thesis in details – average goal

As above, but add:

- Recall how the Jones polynomials arises as a special case of the construction in [RT16].
- Explain how explicit computation can be done using symmetric webs.

The thesis in details – optimal goal

Here are some open question which (if time suffices) deserve further study.

- Can one find a suitable extension of the symmetric web calculus to higher ranks as *e.g.* in [RW20, Section 2.1].
- What is a good basis of the symmetric web calculus, *e.g.*in the sense of [Fon12] or [FKK13]?
- How can the symmetric web calculus work in the non-semisimple case over *e.g.* roots of unities? (See *e.g.* [AST18].)

References

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