LECTURE GEOMETRY AND TOPOLOGY – MATH3061

Disclaimer

Nobody is perfect, and I might have written or said something silly. If there is any doubt, then please check the references or contact me. All questions welcome!

Disclaimer

This document is for the topology part of the course only. The first 6 weeks are on geometry.

What?

The aim of the unit is to expand visual/geometric ways of thinking. The Geometry section is concerned mainly with transformations of the Euclidean plane (that is, bijections from the plane to itself), with a focus on the study of isometries (proving the classification theorem for transformations which preserve distances between points), symmetries (including the classification of frieze groups) and affine transformations (transformations which map lines to lines). The basic approach is via vectors and matrices, emphasizing the interplay between geometry and linear algebra. The study of affine transformations is then extended to the study of collineations in the real projective plane, including collineations which map conics to conics. The Topology section considers graphs, surfaces and knots from a combinatorial point of view. Key ideas such as homeomorphism, subdivision, cutting and pasting and the Euler invariant are introduced first for graphs (1-dimensional objects) and then for triangulated surfaces (2-dimensional objects). Topics include the classification of surfaces, map coloring, decomposition of knots and knot invariants.

Who?

Second semester students in Mathematics interested in a mixture of (linear) geometry and discrete mathematics, but everyone is welcome.

Where and when?

- The lecture.
  - Monday 10:00-11:00, Thursday 09:00-10:00, Friday 09:00-10:00.
  - Starting 7th week, ending 12th week.
  - Carslaw Lecture Theatre 275 and online via zoom.
- The tutorials.
  - Friday 13:00-14:00.
  - Starting 8th week, ending 13th week.
  - Carslaw Seminar Room 350.

Material for the lecture

- There is a script [Hi11] available via Canvas that the lecture will follow.
  Additional literature (not mandatory but recommendations only). The recommended literature from the course outline is [Ad94], [Bl67] and [FiGa91]. The lecture sometimes takes a different perspective and I sometimes borrow the exposition from [Ba10], [BoMu08], [BrHa12], [We96] or [Wi96] for graphs, and from [A+21], [FaSt96] and [Ka93] for surfaces and knots.
- Website www.dtubbenhauer.com/lecture-geotop-2022.html

- Prerecorded lectures on the "What is...algebraic topology?" and "What is...geometric topology?" playlists here:
  www.youtube.com/c/VisualMath/playlists
- One exercise sheet per week; six in total.

Schedule and some details.

7. Basics about graphs – Graphs, subdivision, trees, Eulerian circuits.
   - **Speaker.** Daniel Tubbenhauer.
   - **Plan.** We discuss graphs, and various of their properties. Important topics include subdivision, trees, and Eulerian circuits.
   - **Literature.** [Hi11, Sections 1, 2 and 3].

8. Surfaces I – Various surfaces, homeomorphism, Euler characteristic.
   - **Speaker.** Daniel Tubbenhauer.
   - **Plan.** We discuss basics about the standard surfaces such as discs, annuli, tori, sphere etc. Topics include homeomorphism, stereographic projection, triangulations and Euler characteristic.
   - **Literature.** [Hi11, Sections 4, 5 and 6].

9. Surfaces II – Invariance under subdivision, cutting and pasting, orientation.
   - **Speaker.** Daniel Tubbenhauer.
   - **Plan.** Continuing with surfaces, the topics are now invariance under subdivision, cutting and pasting, boundaries, orientations and edge equations.
   - **Literature.** [Hi11, Sections 7, 8, 9 and 10].

10. Surfaces III – Classification of surfaces.
    - **Speaker.** Daniel Tubbenhauer.
    - **Plan.** The highlight in this section is the classification of surfaces. This includes discussing genus, oriented closed surfaces in three dimensions, handles, and crosscaps. The platonic solids will also play a role.
    - **Literature.** [Hi11, Sections 11, 12 and 13].

11. Graphs and surfaces – Graphs on surfaces, planar graphs.
    - **Speaker.** Daniel Tubbenhauer.
    - **Plan.** Merging the topic of the first week with surfaces, the main point are graphs on surfaces. This includes that K_5 is not planar, various coloring theorems such as the five color theorem and the Heawood estimate for maps on surfaces.
    - **Literature.** [Hi11, Sections 14 and 15].

12. Knots – Knots diagrams, knot coloring, Seifert surfaces.
    - **Speaker.** Daniel Tubbenhauer.
    - **Plan.** In the final week the topic are knots from a combinatorial point of view. Topics include polygonals knots, knots diagrams, the unknot, trefoil knots, figure eight knots, knot coloring, knot determinants, Seifert surfaces.

▶ **Literature.** [Hi11, Sections 16, 17, 18 ,19 and 20].

References

[Ad94] C.C. Adams. The knot book. An elementary introduction to the mathematical theory of knots. Revised reprint of the 1994 original. American Mathematical Society, Providence, RI, 2004. xiv+307 pp.

[A+21] Edited by C. Adams, E. Flapan, A. Henrich, L.H. Kauffman, L.D. Ludwig and S. Nelson. Encyclopedia of knot theory. CRC Press, Boca Raton, FL, [2021], @2021. xi+941 pp.

[Ba10] R.B. Bapat. Graphs and matrices. Universitext. Springer, London; Hindustan Book Agency, New Delhi, 2010. x+171 pp.

[Bl67] D.W. Blackett. Elementary topology. A combinatorial and algebraic approach. Academic Press, New York-London 1967 ix+224 pp.

[BoMu08] J.A. Bondy, U.S.R. Murty. Graph theory. Graduate Texts in Mathematics, 244. Springer, New York, 2008. xii+651 pp.

[BrHa12] A.E. Brouwer, W.H. Haemers. Spectra of graphs. Universitext. Springer, New York, 2012. xiv+250 pp.

[FaSt96] D.W. Farmer, T.B. Stanford. Knots and surfaces. A guide to discovering mathematics. Mathematical World, 6. American Mathematical Society, Providence, RI, 1996.

[FiGa91] P.A. Firby, C.F. Gardiner. Surface topology. Second edition. Ellis Horwood Series: Mathematics and its Applications. Ellis Horwood, New York; distributed by Prentice Hall, Inc., Englewood Cliffs, NJ, 1991. 220 pp.

[Hi11] J. Hillman. Topology. Lecture notes for the Topology component of Geometry and Topology. Available via Canvas.

[Ka93] L.H. Kauffman. Knots and physics. Second edition. World Scientific Publishing Co., Inc., River Edge, NJ, 1993. xiv+723 pp.

[We96] D.B. West. Introduction to graph theory. Prentice Hall, Inc., Upper Saddle River, NJ, 1996. xvi+512 pp.

[Wi96] R.J. Wilson. Introduction to graph theory. Fourth edition. Longman, Harlow, 1996. viii+171 pp.

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