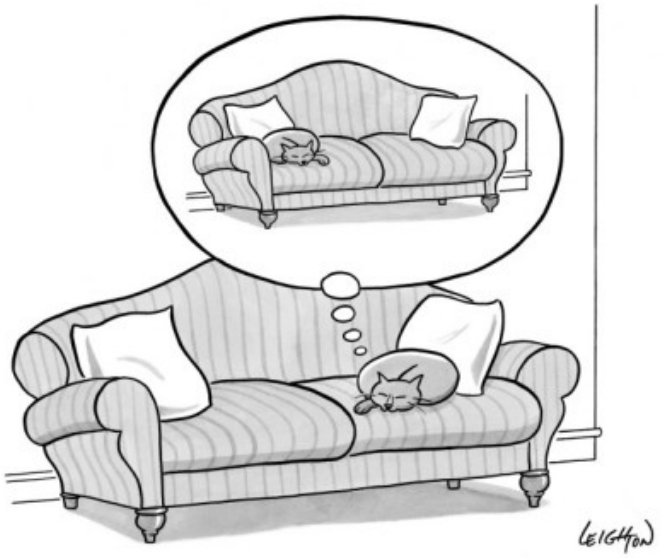
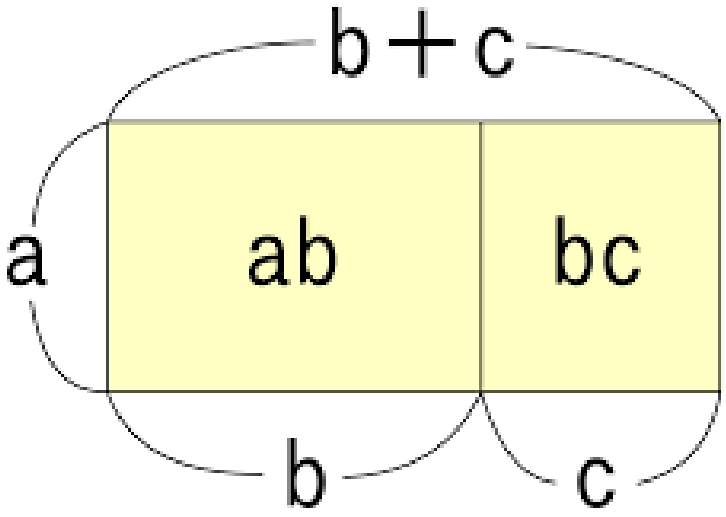
EXERCISES 9: LECTURE CATEGORY THEORY

Exercise 1. A functor $F\colon C \to D$ is a biadjoint of $G\colon D \to C$, if F is the left and right adjoint of G. Give an example of a biadjunction.



What is the cat in the dream dreaming?

Exercise 2. Show that $X \otimes \_\colon \mathbb{K}\text{VECT} \to \mathbb{K}\text{VECT}$ has a right adjoint for all $\mathbb{K}$-vector spaces X. Conclude that $X \otimes (Y \oplus Z) \cong (X \oplus Y) \otimes (X \oplus Z)$.



Hint: Left adjoints preserve colimits.

Exercise 3. Show that the following forgetful functors do not admit adjoints:

$$Forget\colon \text{FIELD} \to \text{cRING}, \quad Forget\colon \text{FIELD} \to \text{RING},$$
$$Forget\colon \text{FIELD} \to \mathbb{Z}\text{MOD}, \quad Forget\colon \text{FIELD} \to \text{SET}.$$

Hint: You can use the adjoint functor theorem.

Exercise 4. Show that $Forget$: GROUP $\to$ MONOID has a left and a right adjoint.

Hint: If M is a monoid, then its group of units $M^\times$ is a group. Moreover, is the universal enveloping group of a monoid (also called noncommutative Grothendieck group) also a group. But existence also follows from abstract nonsense.

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-ct-2022.html.
- The distinction between "large classes" and "small classes (sets)" turns out is crucial for many categorical considerations, but somehow makes the language more cumbersome. If not stated otherwise (which happens rarely and will be easy to spot), then all set-theoretical issues will be strategically ignored in the lecture and on the exercise sheets.
- There might be typos on the exercise sheets, my bad, so be prepared.