

## EXERCISES 8: LECTURE CATEGORY THEORY

**Exercise 1.** Verify that all the functors

- ▶ The free-forget adjunction (F=Forget, G=Free)

X		ℚVECT		MONOID		GROUP		ℤMOD		RING		ℚALG		CAT
Y		SET		SET		SET		SET		SET		SET		QUIVER

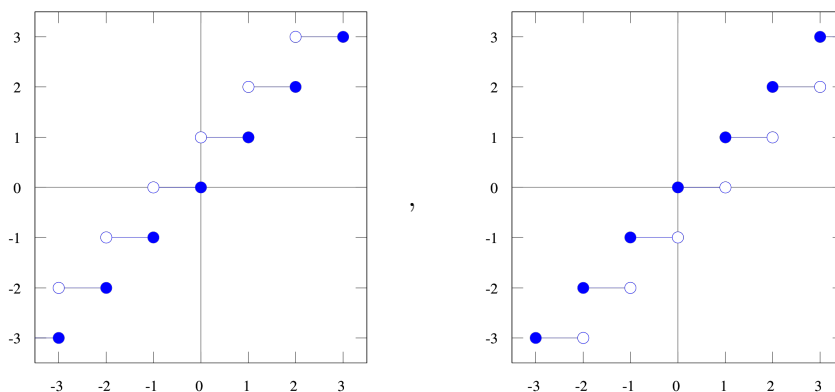
- ▶ X=FIELD, Y=DOMAIN, F=Forget, G=Field of fractions
- ▶ X=ℂVECT, Y=ℝVECT, F=Forget, G=Scalar extension one can vary ℝ, ℂ
- ▶ X=pRING, Y=RING, F=Forget, G=Polynomial ring
- ▶ The tensor-hom adjunction, e.g. currying

$$\text{hom}_C(X \otimes_D Y, Z) \cong \text{hom}_D(X, \text{hom}_C(Y, Z))$$

- ▶ X=RING, Y=GROUP, F=Group of units, G=Group ring
- ▶ X=ℤMOD, Y=GROUP, F=Include, G=Abelianization
- ▶ Any equivalence is an adjoint pair

are adjoint pairs.

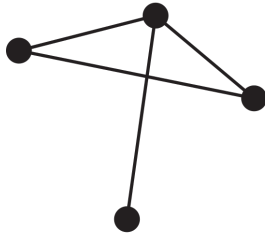
**Exercise 2.** Take  $\mathbb{R}$  with  $x \rightarrow y$  if  $x \leq y$  as a category, and similarly for  $\mathbb{Z}$ . Let  $U: \mathbb{Z} \rightarrow \mathbb{R}$  be the embedding. Show that ceiling and floor are left respectively right adjoints of  $U$ .



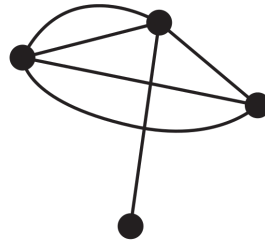
**Exercise 3.** Define left and right adjoints to the object functor  $\text{Ob}: \text{CAT} \rightarrow \text{SET}$ .

**Exercise 4.** Recall that a simple directed graph is a directed graph in which there is at most one edge between any pair of vertices. Let sQUIVER denote the associated category. Show that the embedding  $U: \text{sQUIVER} \rightarrow \text{QUIVER}$  has a left adjoint  $I$ .

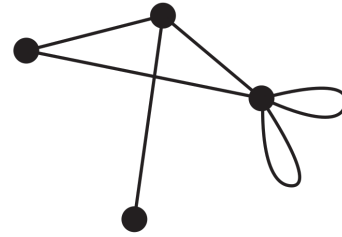
Hint: I should send all of the following graphs to the left-hand one:



*simple graph*



*nonsimple graph  
with multiple edges*



*nonsimple graph  
with loops*

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- ▶ The exercises are optimal and not mandatory. Still, they are highly recommend.
  - ▶ There will be 12 exercise sheets, all of which have four exercises.
  - ▶ The sheets can be found on the homepage [www.dtubbenhauer.com/lecture-ct-2022.html](http://www.dtubbenhauer.com/lecture-ct-2022.html).
  - ▶ The distinction between “large classes” and “small classes (sets)” turns out is crucial for many categorical considerations, but somehow makes the language more cumbersome. If not stated otherwise (which happens rarely and will be easy to spot), then all set-theoretical issues will be strategically ignored in the lecture and on the exercise sheets.
  - ▶ There might be typos on the exercise sheets, my bad, so be prepared.