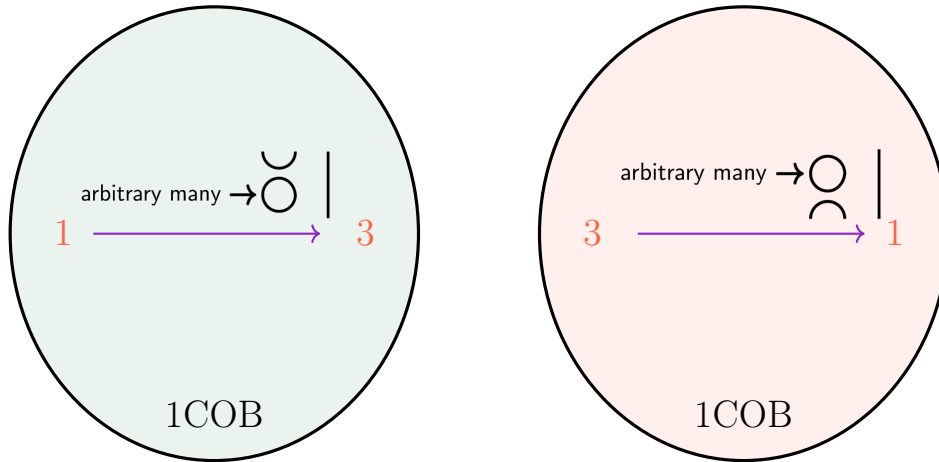
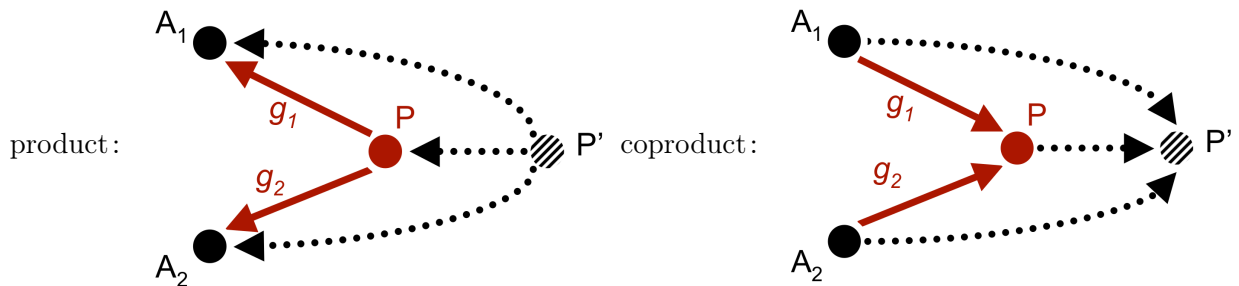


## EXERCISES 6: LECTURE CATEGORY THEORY

**Exercise 1.** Show that 1COB does not have initial and terminal objects.

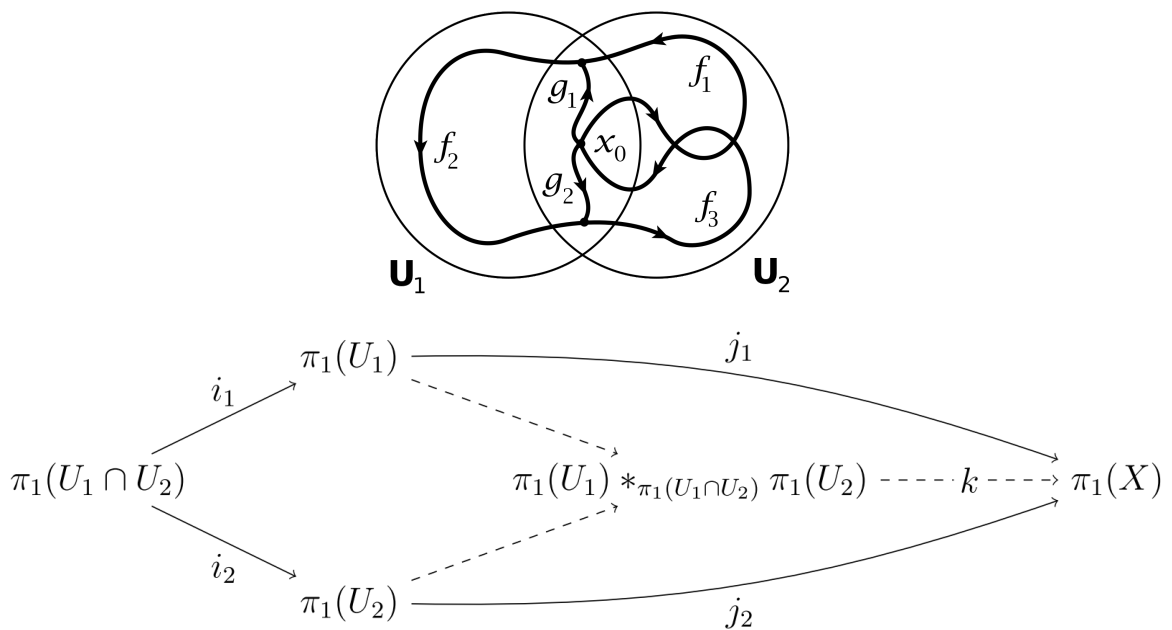


**Exercise 2.** What are (co)products in  $\mathbb{K}\text{VECT}$ ?



What about  $\mathbb{K}\text{fdVECT}$ ?

**Exercise 3.** Explain why and in what sense the Seifert–van Kampen is a pushout.

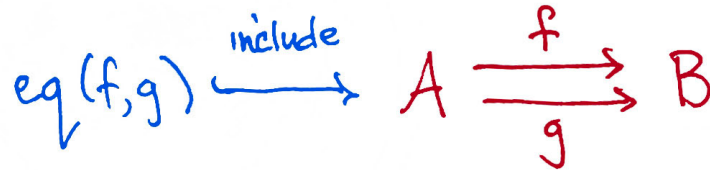


Addendum:

- ▶ Seifert–van Kampen theorem: [https://en.wikipedia.org/wiki/Seifert-Van\\_Kampen\\_theorem](https://en.wikipedia.org/wiki/Seifert-Van_Kampen_theorem).
- ▶ Hint: <https://math.stackexchange.com/questions/1105172/the-seifert-van-kampen-theorem-as-a-push-out>

**Exercise 4.** What of the following exists in the category FIELD?

- a) Initial and terminal objects.
- b) Products and coproducts.
- c) Pushouts and pullbacks.
- d) Equalizers and coequalizers.



= the diagram you start with

Hint: <https://math.stackexchange.com/questions/359352/limits-and-colimits-in-the-category-of-fields>

- ▶ The exercises are optional and not mandatory. Still, they are highly recommend.
- ▶ There will be 12 exercise sheets, all of which have four exercises.
- ▶ The sheets can be found on the homepage [www.dtubbenhauer.com/lecture-ct-2022.html](http://www.dtubbenhauer.com/lecture-ct-2022.html).
- ▶ The distinction between “large classes” and “small classes (sets)” turns out is crucial for many categorical considerations, but somehow makes the language more cumbersome. If not stated otherwise (which happens rarely and will be easy to spot), then all set-theoretical issues will be strategically ignored in the lecture and on the exercise sheets.
- ▶ There might be typos on the exercise sheets, my bad, so be prepared.