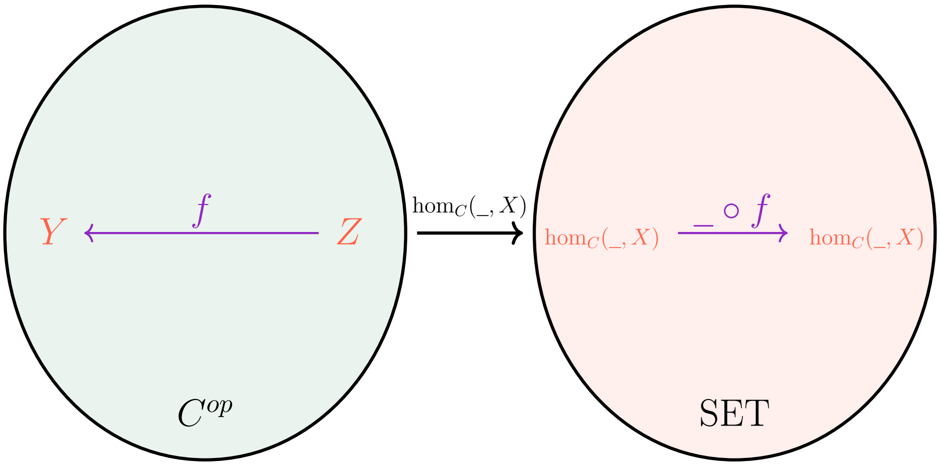
EXERCISES 5: LECTURE CATEGORY THEORY

Exercise 1. Verify that $\hom_C(\_, X)$ is a functor. What would happen if one uses C instead of C^{op}, and $\hom_C(\_, X)$ instead of $\hom_C(X, \_)$?



Exercise 2. Prove Cayley's theorem using the Yoneda lemma.

Addendum:

- Cayley's theorem: https://en.wikipedia.org/wiki/Cayley's_theorem.
- Hint: https://en.wikipedia.org/wiki/Yoneda_lemma#Relationship_to_Cayley's_theorem

Exercise 3. Show that there is precisely one natural transformation $id_{\mathrm{SET}} \Rightarrow id_{\mathrm{SET}}$.

Exercise 4. Make the following statement precise:

The standard descriptions of topological spaces by means of

a) neighborhoods,

b) open sets,

c) closure operators, or

d) convergent filters,

give technically different categories, all of which are concretely isomorphic.

- The exercises are optimal and not mandatory. Still, they are highly recommend.

- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-ct-2022.html.
- The distinction between "large classes" and "small classes (sets)" turns out is crucial for many categorical considerations, but somehow makes the language more cumbersome. If not stated otherwise (which happens rarely and will be easy to spot), then all set-theoretical issues will be strategically ignored in the lecture and on the exercise sheets.
- There might be typos on the exercise sheets, my bad, so be prepared.