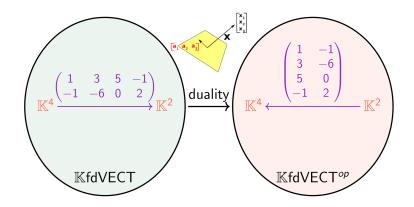# EXERCISES 2: LECTURE CATEGORY THEORY

Exercise 1. Let $\mathbb{K}\mathrm{fdVECT}$ be the category of finite-dimensional $\mathbb{K}$-vector spaces. Describe $\mathbb{K}\mathrm{fdVECT}^{op}$.

Hint:

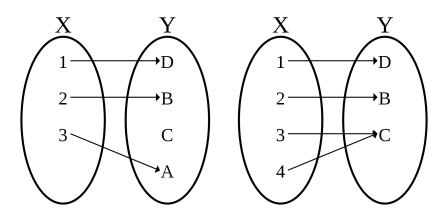**Exercise 2.** For a set X with a preorder relation $<$ let $\mathrm{ORD}(X,<)$ be the category with:

▶ Objects are elements of X.

▶ There is precisely one non-identity arrow $x \to y$ between $x, y \in X$ if $x < y$, and no non-identity arrows otherwise.

▶ Composition is defined using the transitivity of $<$.

Verify that $\mathrm{ORD}(X,<)$ is a category and describe $\mathrm{ORD}(X,<)^{op}$.

Exercise 3. For $f\colon X \to Y$ let $P_C(f)$ be the property:

$$\exists\, g\colon Y \to X \text{ with } X \xrightarrow{f} Y \xrightarrow{g} X = id_X.$$

What is $P_C^{op}(f)$? Describe both, $P_C(f)$ and $P_C^{op}(f)$, in SET.

Hint:

Exercise 4. Fill in the questions marks in the following table.

Name	Monic	Epic	Isos
SET	Injective	Surjective	Bijective
1COB	?	?	?
fSET	?	?	?
pSET	?	?	?
GROUP	?	?	?
TOP	?	?	?
$\mathbb{K}$VECT	?	?	?
$\mathbb{K}$MAT	?	?	?
●	?	?	?

▶ The exercises are optimal and not mandatory. Still, they are highly recommend.

▶ There will be 12 exercise sheets, all of which have four exercises.

▶ The sheets can be found on the homepage www.dtubbenhauer.com/lecture-ct-2022.html.

▶ The distinction between "large classes" and "small classes (sets)" turns out is crucial for many categorical considerations, but somehow makes the language more cumbersome. If not stated otherwise (which happens rarely and will be easy to spot), then all set-theoretical issues will be strategically ignored in the lecture and on the exercise sheets.

▶ There might be typos on the exercise sheets, my bad, so be prepared.