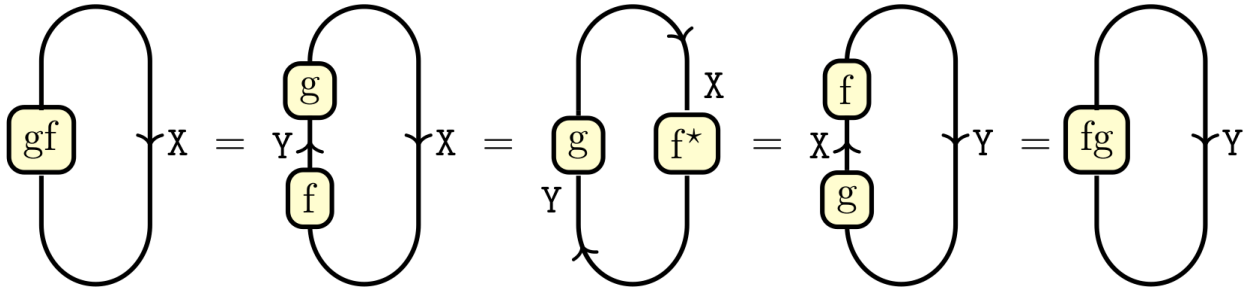


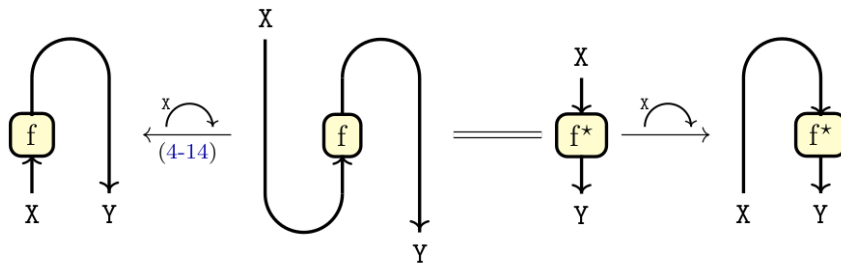
EXERCISES 12: LECTURE CATEGORY THEORY

Exercise 1. Find the statement that is proven by

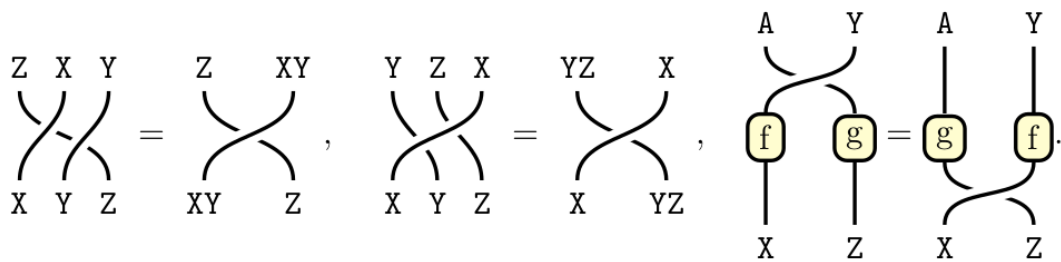


Hint: A trace satisfies $tr(fg) = tr(gf)$.

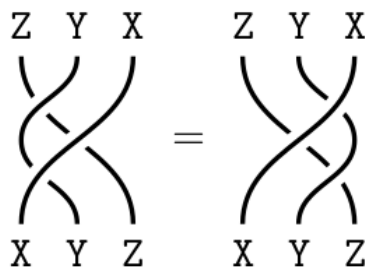
Exercise 2. Explain the diagrammatic proof given by



Exercise 3. In a braided category, show that



imply



Exercise 4. In a pivotal and braided a category we could have the following three relations:

$$(A): \begin{array}{c} X \\ \uparrow \\ \text{loop} \\ \downarrow \\ X \end{array} = \begin{array}{c} X \\ \uparrow \\ | \\ \downarrow \\ X \end{array} = \begin{array}{c} X \\ \uparrow \\ \text{cup} \\ \downarrow \\ X \end{array}, \quad (B): \begin{array}{c} X \\ \uparrow \\ \text{loop} \\ \downarrow \\ X \end{array} = \begin{array}{c} X \\ \uparrow \\ \text{cup} \\ \downarrow \\ X \end{array}, \quad (C): \begin{array}{c} x \\ \uparrow \\ \text{loop} \\ \downarrow \\ x \end{array} = \begin{array}{c} x \\ \uparrow \\ | \\ \downarrow \\ x \end{array} = \begin{array}{c} x \\ \uparrow \\ \text{cup} \\ \downarrow \\ x \end{array}$$

Show that (A) implies (B) and (B) implies (A). What about the converses?

Hint: The converses are false. Try to construct counterexamples.

- ▶ The exercises are optimal and not mandatory. Still, they are highly recommend.
- ▶ There will be 12 exercise sheets, all of which have four exercises.
- ▶ The sheets can be found on the homepage www.dtubbenhauer.com/lecture-ct-2022.html.
- ▶ The distinction between “large classes” and “small classes (sets)” turns out is crucial for many categorical considerations, but somehow makes the language more cumbersome. If not stated otherwise (which happens rarely and will be easy to spot), then all set-theoretical issues will be strategically ignored in the lecture and on the exercise sheets.
- ▶ There might be typos on the exercise sheets, my bad, so be prepared.