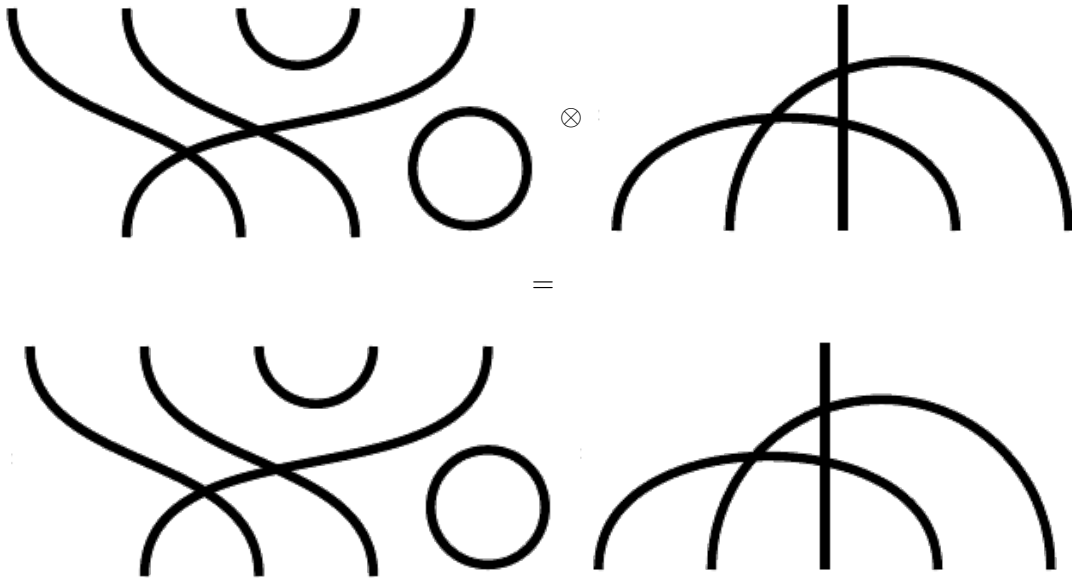# EXERCISES 11: LECTURE CATEGORY THEORY

Exercise 1. Show that 1COB with $\otimes$ being horizontal juxtaposition is a monoidal category.

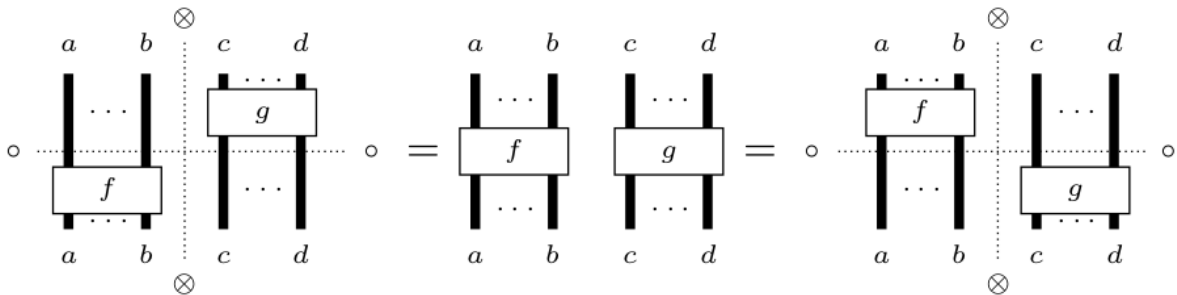**Exercise 2.** Show that $\mathbb{K}$VECT is a monoidal category when using either $\otimes = \otimes$ or $\otimes = \oplus$. Are there any monoidal functors $(\mathbb{K}\text{VECT}, \otimes) \to (\text{VECT}, \oplus)$?

Hint: After strictification, a monoidal functor F is defined by satisfying $F(XY) \cong F(X)F(Y)$.

Exercise 3. Verify that $(id \otimes g) \circ (f \otimes id) = (f \otimes id) \circ (id \otimes g)$ holds in any monoidal category.

Hint: Demystify

Exercise 4. Prove that the monoidal unit of a monoidal category is unique up to isomorphism.

Hint: This should remind you of something, right?

1. Uniqueness of Identity

- In a group G, there is only one identity element.
- Proof: Suppose both e and e' are identities of G. Then,
    1. $ae = a$ for all a in G, and
    2. $e'a = a$ for all a in G.

  Let $a = e'$ in (1) and $a = e$ in (2).

  Then (1) and (2) become

  (1) $e'e = e'$, and (2) $e'e = e$.

  It follows that $e = e'$.

▶ The exercises are optimal and not mandatory. Still, they are highly recommend.

▶ There will be 12 exercise sheets, all of which have four exercises.

▶ The sheets can be found on the homepage www.dtubbenhauer.com/lecture-ct-2022.html.

▶ The distinction between "large classes" and "small classes (sets)" turns out is crucial for many categorical considerations, but somehow makes the language more cumbersome. If not stated otherwise (which happens rarely and will be easy to spot), then all set-theoretical issues will be strategically ignored in the lecture and on the exercise sheets.

▶ There might be typos on the exercise sheets, my bad, so be prepared.