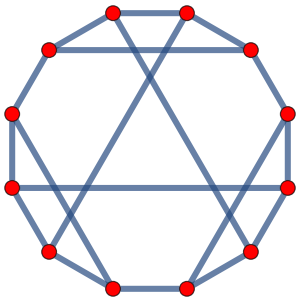
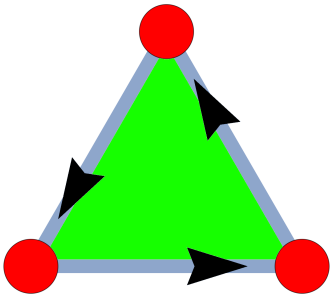
EXERCISES 6: LECTURE ALGEBRAIC TOPOLOGY

Exercise 1. Compute the fundamental group of the cell complex given by the graph



Exercise 2. Find the Cayley graph for $G = \mathbb{Z}/2\mathbb{Z} * \mathbb{Z}/2\mathbb{Z}$. Compute $\pi_1(\mathbb{R}P^2 \vee \mathbb{R}P^2)$ and explain its relation to G.

Exercise 3. Consider the following space X obtained by identifying the three edges of a triangle:

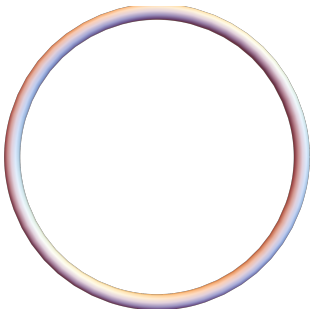
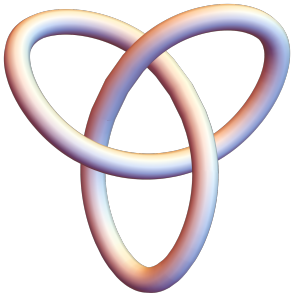
$X =$ 

Compute $\pi_1(X)$. Can you describe a space X' with $\pi_1(X')$ being a fixed but arbitrary finite abelian group?

Addendum:

- Hint: math.stackexchange.com/questions/2834311
- Hint: math.ucr.edu/home/baez/algebraic_topology/Math205B_Mar16.pdf
- Hint: groupprops.subwiki.org/wiki/Classification_of_finite_abelian_groups

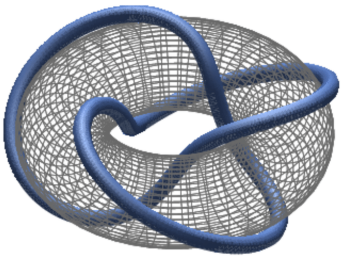
Exercise 4. Take S^1 in $\mathbb{R}^3$ using the unknot embedding Un and the trefoil embedding Tr:

Un =  , Tr = 

Compute $\pi_1(\mathbb{R}^3 \setminus \text{Un})$ and $\pi_1(\mathbb{R}^3 \setminus \text{Tr})$.

Addendum:

- Hint: The trefoil can be embedded into the torus



- Hint: math.stackexchange.com/questions/1774198

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-algtop-2021.html.
- If not specified otherwise, spaces are topological space, maps are continuous *etc.*
- There might be typos on the exercise sheets, my bad, so be prepared.