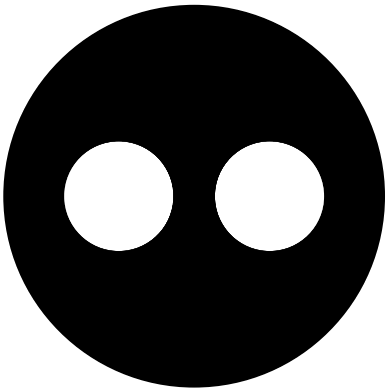
EXERCISES 2: LECTURE ALGEBRAIC TOPOLOGY

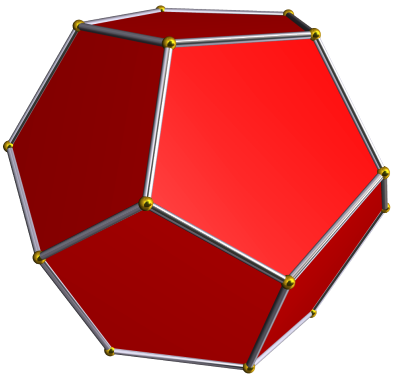
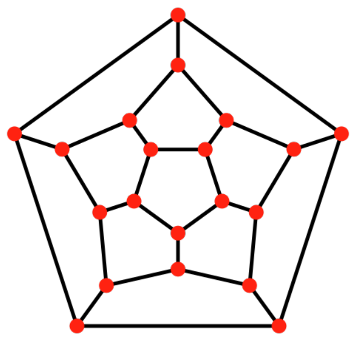
Exercise 1. Construct a cell structure for the disc with two holes D^2_2:

$D^2_2 =$ 

Exercise 2. The Euler characteristic $\chi(P)$ of a polyhedron P is defined by

$$\chi(P) = V - E + F = \#\text{vertices} - \#\text{edges} + \#\text{faces}.$$

1. Compute the Euler characteristic of your favorite platonic solid such as the dodecahedron D:

$D =$  $, D' =$ 

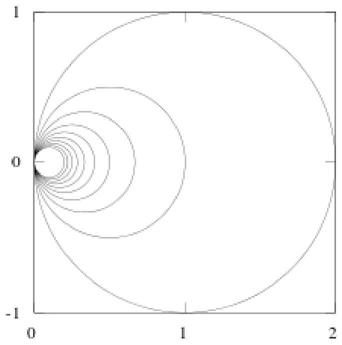
2. What does this has to do with planar graphs? (Graphs that can be drawn on the plane without intersection, *e.g.* D' above.) en.wikipedia.org/wiki/Planar_graph

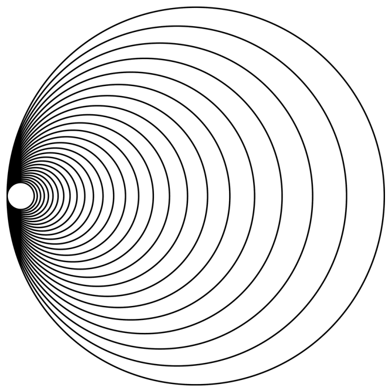
Exercise 3. Construct a deformation retraction of the punctured torus onto the space given by two circles intersecting in one point.

Addendum:

- In formulas and using complex numbers this means a deformation retract from $(S^1 \times S^1) \setminus \{(-1,-1)\}$ onto $(S^1 \times \{i\}) \cup (\{1\} \times S^1)$.
- Hint: www.youtube.com/watch?v=j2HxBUaoaPU

Exercise 4. The Hawaiian earrings H is the following subset of $\mathbb{R}^2$, with the induced topology:

$H =$ 

another illustration: 

1. Show that H is not homeomorphic to $\bigvee_{\mathbb{N}} S^1$.
2. Show that H is not homotopy equivalent to $\bigvee_{\mathbb{N}} S^1$.
3. Can H be realized as a cell complex (meaning is it homotopy equivalent ot a cell complex)?

Addendum:

- Formally, H is *e.g.* the union of circles of radius $1/n$ and midpoint $(1/n, 0)$.
- Note that 3. $\Rightarrow$ 2. $\Rightarrow$ 1. (Can you see why?)
- Hint: math.stackexchange.com/questions/523416

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-algtop-2021.html.
- If not specified otherwise, spaces are topological space, maps are continuous *etc.*
- There might be typos on the exercise sheets, my bad, so be prepared.