Categorification of the virtual Jones polynomial

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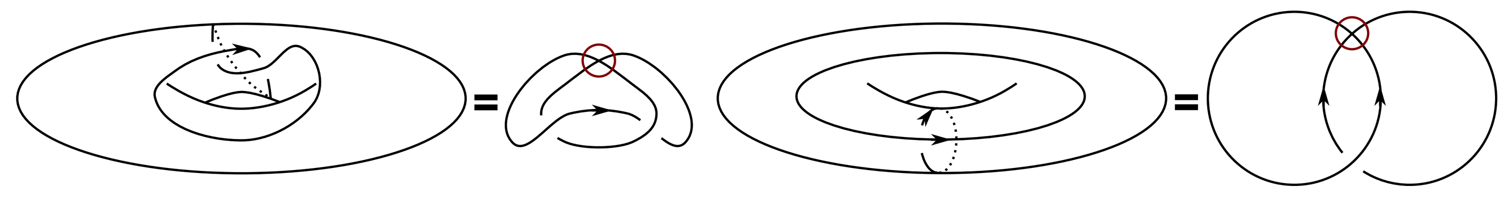
Virtual links

- Virtual link diagrams L_D are a combinatorial description of link diagrams in F_g;



The virtual trefoil knot and the virtual Hopf link.

- but there are much more virtual links than classical links;

	$n \leq 3$	$n = 4$	$n = 5$	$n = 6$
classical	2	3	5	8
virtual	8	109	2448	90235

The number of different knots with n crossings.

- virtual links are a new concept by L. Kauffman (see [2]) and yield a complicated combinatorial structure. Every invariant is helpful!
- the approach is to categorify the virtual Jones polynomial using a variant of Khovanov homology.

The algebraic complex

To obtain an algebraic complex we use a certain kind of topological quantum field theory (TQFT), an unoriented TQFT $\mathcal{F}$. These uTQFTs are in 1:1 correspondence with skew-extended Frobenius algebras.

Theorem. *Let $\mathcal{F}$ be an aspherical uTQFT. Then the algebraic complex $\mathcal{F}(⟦\cdot⟧)$ is an invariant of virtual links iff the corresponding skew-extended Frobenius algebra can be obtain from a certain universal skew-extended Frobenius algebra $\mathcal{F}_U$ through base change. The graded Euler characteristic of one of these complexes is the virtual Jones polynomial.*

Summary

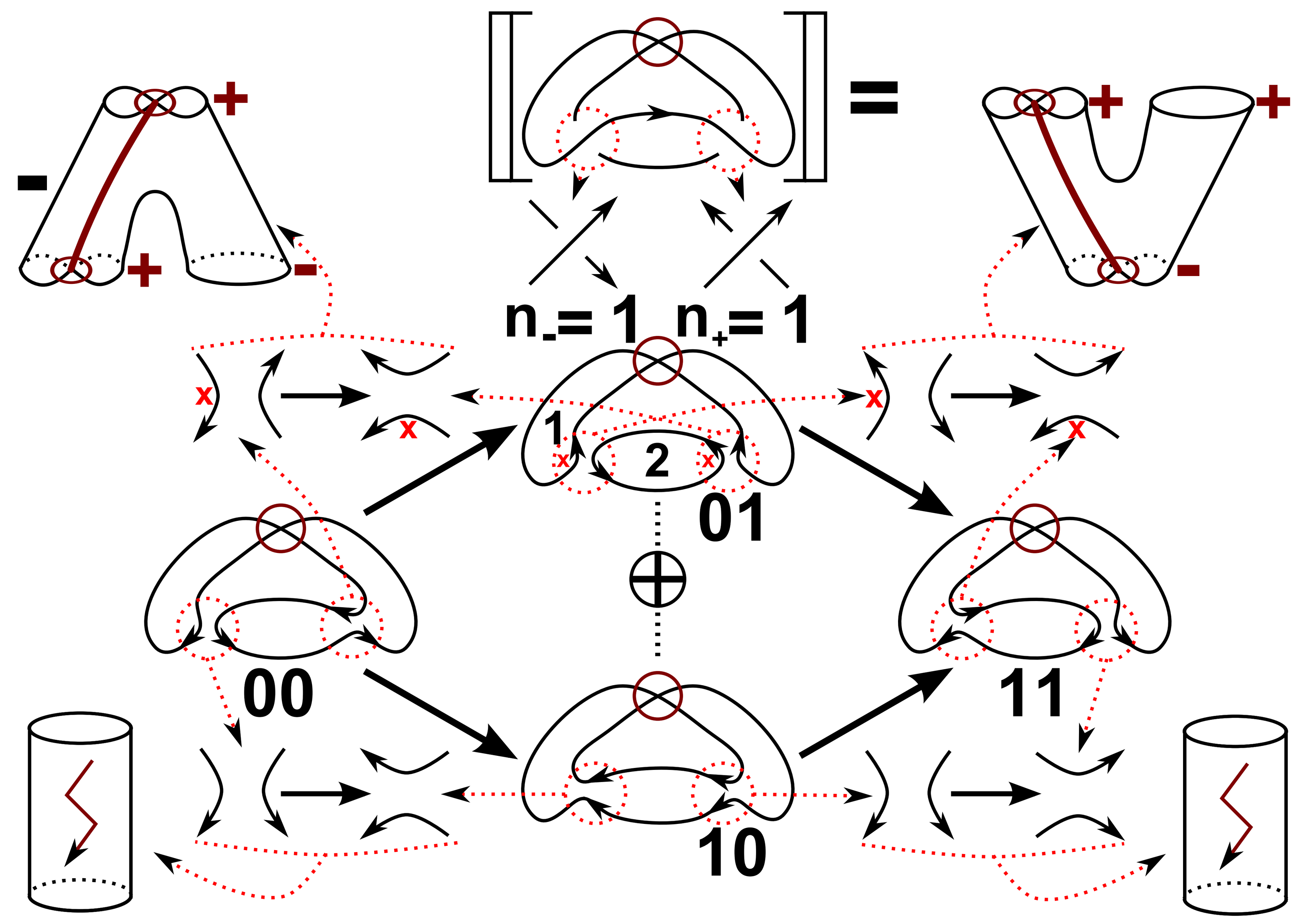
Computations with our MATHEMATICA program vKH.m show that our invariant is strictly stronger than the virtual Jones polynomial. Furthermore our categorification also works for virtual tangles and is related to different other invariants, like the Rasmussen invariant, odd Khovanov homology and the $\mathfrak{sl}_n$-polynomials.

The geometric complex

To define the categorification we defined a special geometric category $\mathbf{uCob}^2_R(\emptyset)$, i.e. a category of cobordisms between v-link resolutions in the spirit of D. Bar-Natan (see [1]), but the morphisms are possible unorientable cobordisms immersed into $\mathbb{R}^2 \times [-1,1]$ together with a decoration $+,-$.

The geometric chain complex $⟦L_D⟧$ for a virtual link diagram L_D with n classical crossings is defined purely combinatorial. The complex itself is a n-dim hypercube whose vertices are resolutions of the diagram L_D and whose edges are saddle cobordisms between the resolutions.

There is a certain and very important rule how to spread signs and decorations to the cobordisms.



The complex of a virtual diagram of the unknot. The lower surfaces are a two times punctured $\mathbb{RP}^2$.

Theorem. *The geometric complex $⟦L_D⟧$ is a well-defined chain complex whose homotopy class is an invariant of virtual links up to some local relations.*

References

1. D. Bar-Natan, Khovanov's homology for tangles and cobordisms, *Geo. and Top.*, **9** (2005), 1443–1499
2. L. Kauffmann, Virtual knot theory, *Math. Notes*, **20** (1999), 663–690
3. D.Tubbenhauer, Khovanov homology for virtual links using cobordisms, *preprint*, arXiv:1111.0609v2 (2011)

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