EXERCISES 7: LECTURE FOUNDATIONS OF MATHEMATICS

Exercise 1. Show that for $m, n \in \mathbb{N}_0$ with $m \leq n$ one has $(m!(n-m)!)|n!$.

Exercise 2. Show the following statements.
  (a) There are infinitely many primes.
       (Hint: $p_1 p_2 \cdots p_r + 1$ is not divisible by $p_1, p_2, \ldots, p_r$.)

  (b) The are arbitrarily big gaps between primes.
       (Hint: which of $(n+1)! + 2, (n+1)! + 3, \ldots, (n+1)! + n, (n+1)! + n + 1$ are primes?)

Exercise 3. Define a map $f \colon \mathbb{N}_0 \setminus \{0\} \to \mathbb{N}_0$ recursively via
$$f(1) = 2, \quad f(n) = f(n-1) + n2^n, \quad \forall n > 1.$$
Show that $f(n) = (n-1)2^{n+1} + 2$ for all $n \in \mathbb{N}_0 \setminus \{0\}$.

Exercise 4. Show that there exist $n!$ total orders on a set with n elements.

Submission of the exercise sheet: 12.Nov.2018 before the lecture. **Return of the exercise sheet**: 22.Nov.2018 during the exercise sessions.