## **EXERCISES 4: LECTURE FOUNDATIONS OF MATHEMATICS**

**Exercise 1.** Let  $X = \{a, b, c\}$ . List all possible equivalence relations on X.

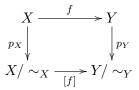
**Exercise 2.** Let X be a set, and let  $X^X$  denote the set of all maps  $X \to X$ . Further, let S(X) denote the set of all bijective maps  $X \to X$ . Show:

- (a) If  $f, g \in S(X)$ , then  $g \circ f$  and  $f \circ g$  are also in S(X).
- (b) If X has at least two elements, then  $X^X$  is not commutative with the operation given by  $\circ$ .
- (c) If X has at least three elements, then S(X) is not commutative with the operation given by  $\circ$ .

**Exercise 3.** Let X, Y be sets and let  $\sim_X, \sim_Y$  be equivalence relations on these sets. Moreover, let  $f: X \to Y$  be a map such that

$$(\star): \qquad (x_1 \sim_X x_2) \Rightarrow (f(x_1) \sim_Y f(x_2)) \quad \forall x_1, x_2 \in X.$$

Show that there is a unique map [f] such that



commutes. What happens if  $(\star)$  in the case where  $\sim_X$  is the identity relation? (Meaning that  $(x_1 \sim_X x_2) \Leftrightarrow (x_1 = x_2)$ .)

**Exercise 4.** Let  $(X, \leq)$  be an ordered set. Further, let A and B subsets of X which are bounded above. Show the following statements in the case where the corresponding suprema und infima exist:

- (a)  $\sup(A \cup B) = \sup(\sup(A), \sup(B)).$
- (b) If  $A \subset B$ , then  $\sup(A) \leq \sup(B)$ .
- (c) If  $A \cap B \neq \emptyset$ , then  $\sup(A \cap B) \leq \inf(\sup(A), \sup(B))$ .

Formulate and prove the corresponding statements for subsets C and D of X which are bounded below.

Submission of the exercise sheet: 22.Oct.2018 before the lecture. Return of the exercise sheet: 25.Oct.2018 during the exercise classes.