EXERCISES 12: LECTURE FOUNDATIONS OF MATHEMATICS

Exercise 1. Show that $\mathbb{Q}$ is the smallest field contained in $\mathbb{R}$. That is, for all fields $K \subset \mathbb{R}$ one has $\mathbb{Q} \subset K$.

Exercise 2. Let $A, B \neq \emptyset$ be subsets of $\mathbb{R}_{>0}$. Define $A + B = \{a + b \mid a \in A, b \in B\}$ and $A \cdot B = \{ab \mid a \in A, b \in B\}$.

(a) Show that $\sup(A + B) = \sup(A) + \sup(B)$.

(b) Show that $\inf(A + B) = \inf(A) + \inf(B)$.

(c) Show that $\sup(A \cdot B) = \sup(A)\sup(B)$.

(d) Show that $\inf(A \cdot B) = \inf(A)\inf(B)$.

(e) Decide (with a proof) whether (a)-(d) also hold in case $A, B \neq \emptyset$ are subsets of $\mathbb{R}$ instead of $\mathbb{R}_{>0}$.

Exercise 3. Let $A \subset \mathbb{R}$ be a set such that $\inf(A) > 0$. Define $1/A = \{1/a \mid a \in A\}$. Show that $\sup(1/A) = 1/(\inf A)$.

Exercise 4. Show that $\mathbb{R}$ is not countable.

Submission of the exercise sheet: 17.Dec.2018 before the lecture. **Return of the exercise sheet**: 20.Dec.2018 during the exercise sessions.