EXERCISES 1: LECTURE FOUNDATIONS OF MATHEMATICS

Exercise 1. Are the following conclusions logically sound? Justify your answer.

(a) If the consensus of the mayor does not change, then there will not be a swing in the political mood of the village Seldwyla. But if the mayor becomes more consensual, then Seldwyla will join the Ennettaler Union. In this case their will be an economic upswing, and Seldwyla will become a village where milk and honey flow around. On the other hand, if the political mood does not change, then there will be a recession.

Thus, there will be a recession, or Seldwyla will become a village where milk and honey flow around.

(b) If Vitalstatistix does not neglect his duty, then our well-known Gauls prepare the next wild boar feast. If he neglects his duty, there is too little consumption of warm beer. But it is either enough warm beer drunk or too little. However, the latter is never the case.

Hence, Vitalstatistix never neglects his duty.

Formulate (a) and (b) in terms of logical operators.

Exercise 2. Consider the following atomic formulas.

$$A: \text{ It's snowing.} \qquad B: \text{ It's freezing cold.}$$

Express the sentences below in terms of the atomic formulas defined by A and B. Moreover, specify the corresponding truth tables for each of these expressions.

(a) It's freezing cold and it's snowing.

(b) It's freezing cold, but it's not snowing.

(c) It is not freezing cold and it is not snowing.

(d) Either it is snowing or it is freezing cold.

(e) Either it is snowing or it is freezing cold, but it does not snow when it's freezing cold.

Exercise 3. Formulate for the following logical expressions the corresponding truth tables, where A, B are atomic formulas.

(a) $(A \wedge (\neg B)) \vee A$.

(b) $(A \vee (\neg B)) \wedge A$.

(c) $(A \wedge (\neg B)) \vee (\neg A)$.

(d) $(A \vee (\neg B)) \wedge (\neg A)$.

(e) $(A \wedge B) \wedge ((\neg A) \vee (\neg B))$.

Are there tautologies (expressions which are always true) or contradictions (expressions which are always false) among these?

Exercise 4. Decide which of the following logical expressions are equivalent, where A, B, C are atomic formulas.

(a) Expressions:

(i) $\neg(A \wedge B \wedge (\neg C))$.

(ii) $\neg(A \wedge (\neg B)) \Rightarrow ((\neg A) \vee (B \Rightarrow C))$.

(b) Expressions:

(i) $(\neg A) \Leftrightarrow (B \Rightarrow C)$.

(ii) $\neg(A \Rightarrow (\neg B)) \vee C$.

Justify your answer using truth tables.

▷ **Submission of the exercise sheet**: 01.Oct.2018 before the lecture. **Return of the exercise sheet**: 04.Oct.2018 in the exercise classes.

▷ The exercise groups will take place every Thursday from the 27.Sep.2018 (first) to the 20.Dec.2018 (last). There won't be any exercise groups on the 15.Nov.2018.

▷ There will be 12 exercise sheets, all of which have four exercises. Each exercise gives 10 marks. There will be a thirteenth sheet, but I won't be graded.

▷ You need 240 marks to be admitted to the exam. All exercise sheets will be relevant for the exam.

▷ The sheets can be found on the homepage

`http://www.dtubbenhauer.com/lecture-GdM-2018.html`

You only need to hand in one sheet per week.

▷ Write your name clearly on your solutions and staple them together. You can hand in your solutions alone or in pairs.

▷ There might be typos on the exercise sheets, so be prepared.